Problem A (folklore)

Let Γ be a finite undirected graph (without loops or multiple edges). Denote the set of vertices by V. Assume that there are a function $f : V \to \mathbb{Z}$ and a positive integer n such that

$$\sum_v \left| \sum_w (f(v) - f(w)) \right| = 2n,$$

where v runs over all the vertices of Γ and w over all the neighbours of v. Show that there are an $m \le n$ and a collection of m edges such that the graph obtained from Γ by removing those edges is not connected.

Problem B (folklore)

Let ϵ be a positive real number. Show that there is a finite group G that is not a 2-group, but in which the proportion of elements that have 2-power order is at least $1 - \epsilon$.

Problem C (proposed by Hendrik Lenstra)

Let B be a commutative ring and A a subring of B. Assume that the additive group of A has finite index in B. Show that the unit group of A has finite index in the unit group of B.
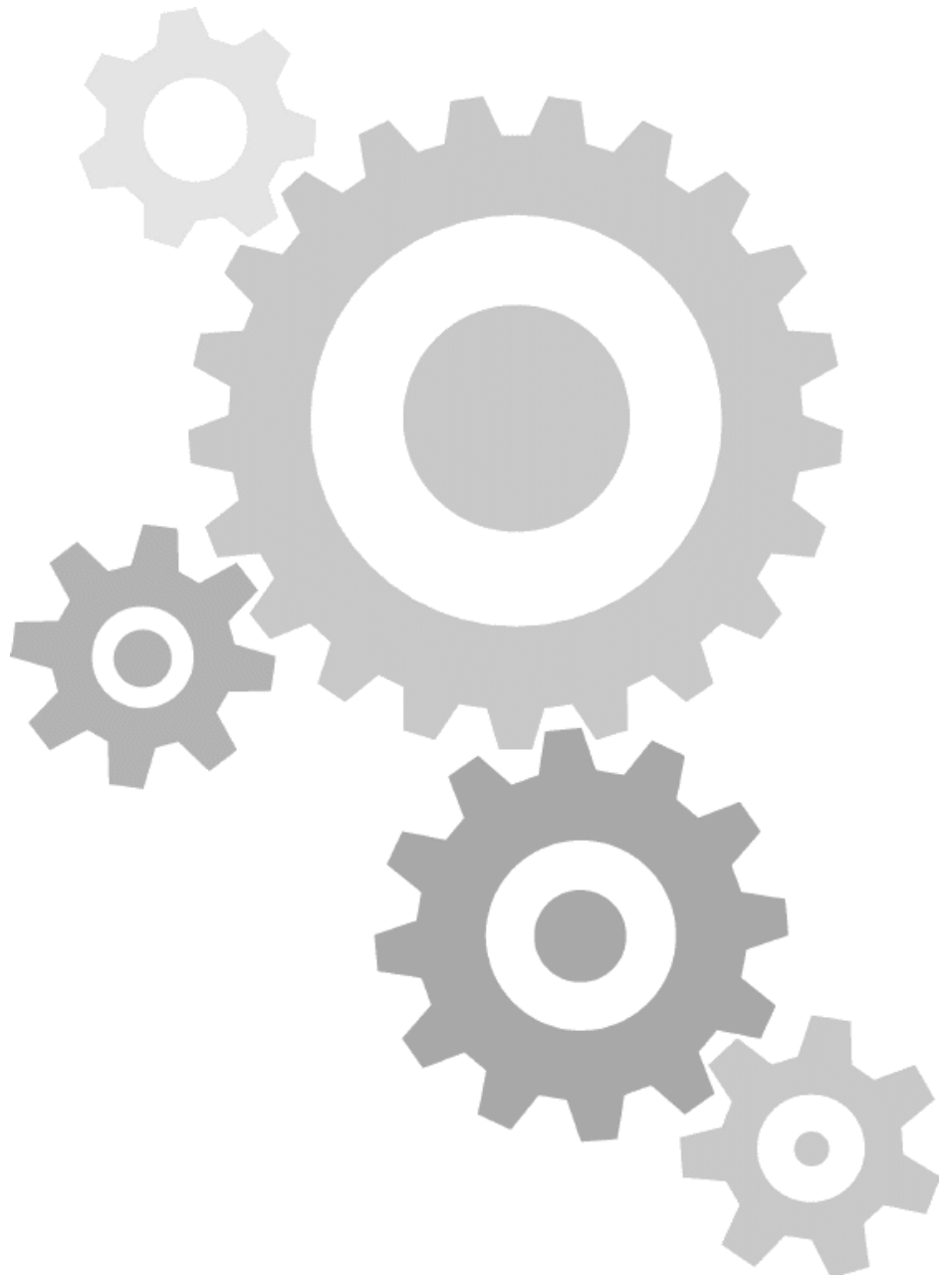
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