Problem Section

Redactie: Johan Bosman Gabriele Dalla Torre Ronald van Luijk Lenny Taelman Problemenrubriek NAW

Mathematisch Instituut Universiteit Leiden Postbus 9512 2300 RA Leiden problems@nieuwarchief.nl www.nieuwarchief.nl/problems Problem A (proposed by Hendrik Lenstra)

Prove that every commutative ring with identity having at most five ideals is a principal ideal ring.

Problem B (proposed by Gabriele Dalla Torre, inspired by the Dutch Mathematical Olympiad 2010) Let $(a_n)_{n\geq 1}$ be a sequence of integers that satisfies

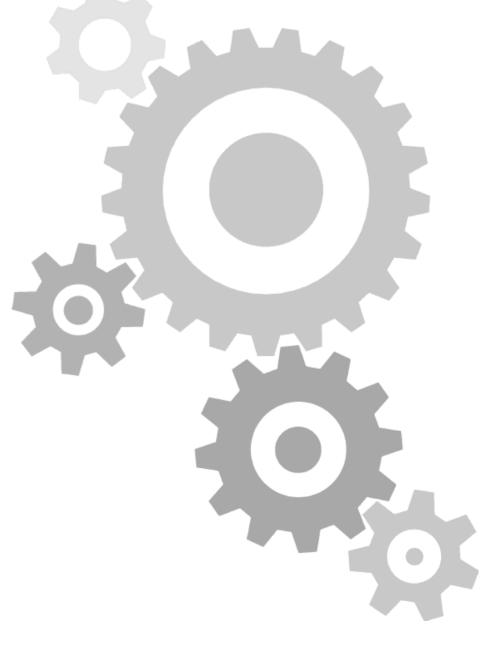
 $a_n = a_{n-1} - \min(a_{n-2}, a_{n-3})$

for all $n \ge 4$. Prove that for every positive integer k there is an n such that a_n is divisible by 3^k .

Problem C (proposed by Rik Bos) Find all positive integers N, r, p with p prime that satisfy

$$\prod_{\ell \leq N} \ell = p(p^r + 1)$$

where the product runs over all primes ℓ not exceeding N.



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