Problem Section

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49

Problemenrubriek NAW Mathematisch Instituut Postbus 9512, 2300 RA Leiden problems@nieuwarchief.nl www.nieuwarchief.nl/problems The deadline for solutions to the problems in this edition is September 1.

**Problem A** (proposed by Gabriele Dalla Torre) Show that for every positive integer *n* and every integer  $m \ge 2$ , we have

$$\sum_{\substack{1 \le i \le n \\ m \nmid i}} \lfloor \log_m(n/i) \rfloor = \lfloor n/m \rfloor.$$

Problem B (folklore)

Let  $b \ge 2$  be an integer. We let  $\sigma_b(n)$  denote the sum of the digits in base b of the integer n. Show that we have

$$\lim_{n \to \infty} \sigma_b(n!) = \infty.$$

## Problem C (folklore)

Two players play a game of *n*-in-a-row on an infinite checkerboard. The first player plays with white pieces, the second with black pieces. On each move they place one piece on an empty square. The first player to have *n* consecutive pieces in a row or column wins. For which values of *n* is there a winning strategy for one of the players?

