

# Problemen

| Problem Section

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**Problem A** (folklore)

Let  $k$  be a non-negative integer. Let  $S \subset \mathbf{Z}$  be a set consisting of  $2^{k+1} - 1$  integers. Show there exists a subset  $T \subset S$  of cardinality  $2^k$  such that the sum of the elements of  $T$  is divisible by  $2^k$ .

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**Problem B** (folklore)

Find all functions  $f : \mathbf{R}_{>0} \rightarrow \mathbf{R}_{>0}$  such that

$$f(x + y) \geq f(x) + yf(f(x))$$

for all  $x$  and  $y$  in  $\mathbf{R}_{>0}$ .

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**Problem C** (folklore)

Let  $V$  be an infinite-dimensional vector space. Show that the dimension of the dual space  $V^*$  equals the cardinality of  $V^*$ .

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