# Problemen

| Problem Section

**Problem A** (folklore)
Let $s$ be a real number. Find all continuous functions $f\colon \mathbf{R}_{>0} \to \mathbf{R}_{>0}$ that satisfy

$$f(xy) = f(x)^{y^s} f(y)^{x^s}$$

for all $x$ and $y$.

**Problem B** (proposed by Lee Sallows)
In the accompanying picture, nine numbered counters occupy the cells of a $3 \times 3$ board so as to make a magic square. They form 8 collinear triples, and each triple yields the same sum 15.

| | | |
|---|---|---|
| 8 | 1 | 6 |
| 3 | 5 | 7 |
| 4 | 9 | 2 |

Place nine counters, numbered 1 through 9, on the same board, again one in each cell, so that they form 8 collinear triples, now showing a common sum of 16 rather than 15.

**Problem C** (folklore)
If $n$ is a nonnegative integer, define $a(n)$ to be the number of decimal digits of $2^n$ that are larger than or equal to 5. For example, $a(8) = 2$. Evaluate the infinite sum

$$\sum_{i=0}^{\infty} \frac{a(n)}{2^n}.$$

**Problem *** (proposed by Ronald van Luijk)
Prove or disprove that for each rational number $r$ there exist rational numbers $a, b, c, d$ such that

$$r = \frac{a^4 - b^4}{c^4 - d^4}.$$

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