Problemen

| Problem Section

Problem A (folklore)

If x is a real number then we denote by $\lfloor x \rfloor$ and $\lceil x \rceil$ the largest integer smaller than or equal to x and the smallest integer larger than or equal to x, respectively. Prove or disprove: for all positive integers n we have

$$\left\lceil \frac{2}{2^{1/n}-1} \right\rceil = \left\lfloor \frac{2n}{\log(2)} \right\rfloor.$$

Problem B (folklore)

Let (a_i) be a sequence of positive real numbers such that

$$\lim_{n\to\infty} \frac{a_1 + a_2 + \cdots + a_n}{n} = a$$

for some real number a. Show that

$$\lim_{n\to\infty} \frac{a_1a_2 + a_1a_3 + \cdots + a_{n-1}a_n}{n^2} = \frac{a^2}{2},$$

where the numerator on the left is the sum $\sum_{1\le i,j\le n, i<j} a_ia_j$.

Problem C (proposed by Hendrik Lenstra)

Let x be a real number, and m and n positive integers. Show that there exist polynomials f and g in two variables and with integer coefficients, such that

$$x = \frac{f\left(x^n, (1-x)^m\right)}{g\left(x^n, (1-x)^m\right)}.$$

Problem * (see 2008/1-B below)

Let $n > 1$ be an integer. Let S be a set consisting of n integers such that for every $s \in S$ there exist $a, b \in S$ with $s = a + b$. Prove or disprove that there exists a subset $T \subset S$ of cardinality at most $n/2$ whose elements add up to zero.

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