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Problem Section

Problem A (proposed by Hendrik Lenstra)

Let *a* and *b* be integers. Show that the following are equivalent:

- *n* divides $a^n b^n$ for infinitely many positive integers *n*,
- $|a-b| \neq 1$.

Problem B (based on a problem by Hendrik Lenstra)

Let *G* be a finite group and *a* be an element of *G*. Show that the number of elements $g \in G$ that satisfy both $ga \neq ag$ and $ga^2 = a^2g$ is divisible by 4.

Problem C (folklore)

Let *p* be an odd prime number and *A* and *B* two $n \times n$ matrices with entries in **Z** such that $A^p = B^p = 1$, and such that the rank of A - B is 1. Show that $n \ge p$.

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