

# Problemen

| Problem Section



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**Problem A** (Folklore)

Let  $a$  be an integer. Let  $(x_n)_n$  be the sequence determined by  $x_1 = a$  and  $x_{n+1} = 2x_n^2 - 1$ . Show that  $n$  and  $x_n$  are coprime for all  $n$ .

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**Problem B** (Folklore)

Let  $G$  be a group with  $n$  elements and  $S \subset G$  a non-empty subset. Show that the set  $S^n := \{s_1 s_2 \cdots s_n \mid s_i \in S\}$  is a subgroup of  $G$ .

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**Problem C** (Folklore)

(a) Given 2007 points in the plane such that no pair has distance strictly less than one, show that one can find a subset of 288 points in which no pair has distance strictly less than  $\sqrt{3}$ .

(b)\* Supposedly the number 288 in part (a) is not optimal. Find upper and lower bounds for the optimal value.

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