△ Optiver
DERIVATIVES TRADING

Problem A (Proposed by Vietnam for the International Mathematic Olympiad)

1. Find the largest number c such that all natural numbers n satisfy $n\sqrt{2} - \lfloor n\sqrt{2} \rfloor \geq \frac{c}{n}$.
2. For this c, find all natural numbers n such that $n\sqrt{2} - \lfloor n\sqrt{2} \rfloor = \frac{c}{n}$.

Problem B (Proposer known to the editors of NAW)

Find polynomials $f(x)$ and $g(x)$ such that

$$\int_0^x \frac{6t\,\mathrm{d}t}{\sqrt{t^4 + 4t^3 - 6t^2 + 4t + 1}} = \log\left(f(x) + g(x)\sqrt{x^4 + 4x^3 - 6x^2 + 4x + 1} \right).$$

Problem C (based on the IBM Research February 2007 Challenge)

Consider the following game with persons A and B. Player A receives a random number uniformly distributed between 0 and 1. Player B receives two random numbers uniformly distributed between 0 and 1, and chooses the highest one. Each player can then choose to discard his number and receive a new random number between 0 and 1, in order to get a higher number. This choice is made without knowing the other player's number or whether the other player chose to replace his number. The player with the highest number wins. What strategy should the players follow to ensure they will win the game? What is the probability that person B wins the game? See also http://domino.research. ibm.com/Comm/wwwr_ponder.nsf/challenges/February2007.html