Problemen

| Problem Section

Problem A (Proposed by John Scholes)

Define the sequence $\{u_n\}$ by $u_1 = 1$, $u_{n+1} = 1 + (n/u_n)$. Prove or disprove that

$$u_n - 1 \ < \sqrt{n} \leq u_n.$$

Problem B (Folklore)

Given a non-degenerate tetrahedron (whose vertices do not all lie in the same plane), which conditions have to be satisfied in order that the altitudes intersect at one point?

Problem C (Proposed by Jan Draisma)

Let e be a positive integer, and let d be an element of $\{0, 1, 2, \ldots, 3e\}$. Show that the polynomial

$$P \ = \sum_{a \geq 0, b \geq 0, c \geq 0, a+b+c=d} \frac{d!}{a!b!c!} \binom{e}{a} \binom{e}{b} \binom{e}{c} x^a y^b z^c$$

in the three variables x, y, and z is not divisible by $x + y + z$ unless $d = 1$.

Problem * (Proposed by Farideh Firoozbakht)

For a positive integer n, $\sigma(n)$ is defined as the sum of the divisors of n, including n, and $\phi(n)$ is the Euler phi (or totient) function. Let $\psi(n) = \sigma(n)/\phi(n)$. For $n = 2, 3, 6, 12, 14$ and 15, $\psi(n)$ is an integer (respectively $3, 2, 6, 7, 4$, and 3). Does there exist, for every integer k, an integer n such that $\psi(n) = k$?

As a reference, see: http://www.research.att.com/~njas/sequences/A055234

Eindredactie: Matthijs Coster
Redactieadres: Problemenrubriek NAW
Mathematisch Instituut
Postbus 9512
2300 RA Leiden
uwc@nieuwarchief.nl