Problemen

| Problem Section

Problem A (Folklore)
Seventeen students play in a tournament featuring three sports: badminton, squash, and tennis. Any two students play against each other in exactly one of the three sports. Show that there is a group of at least three students who compete amongst themselves in one and the same sport.

Problem B (Proposed by Arthur Engel)
The sequence $\{a_n\}_{n\geq 1}$ is defined by

$$a_1 = 1; a_2 = 12; a_3 = 20; a_{n+3} = 2a_{n+2} + 2a_{n+1} - a_n \ (n \in \mathbf{N}).$$

Prove that $4a_na_{n+1} + 1$ is a square for all $n \in \mathbf{N}$.

Problem C (Proposed by Michiel Vermeulen)
Let G be a finite group of order $p+1$ with p a prime. Show that p divides the order of $Aut(G)$ if and only if p is a Mersenne prime, that is, of the form $2^n - 1$, and G is isomorphic to $(\mathbf{Z}/2)^n$.

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