Problemen

| Problem Section

Problem A (Problem Exchange)

Evaluate
$$\int_0^1 \frac{\log(x+1)}{x^2+1}\mathrm{d}x.$$

Problem B (Proposed by Peter Montgomery)

1. Find an integer n (as small as possible) and rational numbers $a_1, \ldots, a_n$ such that
$$\sqrt{10+3\sqrt{11}} = \sum_{i=1}^{n} \sqrt{a_i}.$$

2. Find an integer n (as small as possible) and rational numbers $b_1, \ldots, b_n$ such that
$$\sqrt[3]{\sqrt[3]{2}-1} = \sum_{i=1}^{n} \sqrt[3]{b_i}.$$

Problem C (Proposed by Paul Penning)

Consider a triangle ABC inscribed in an ellipse. For given A the other vertices can be adjusted to maximize the circumference. Prove or disprove that this maximum circumference is independent to the position of A on the ellipse.

Problem * (Proposed by Christiaan van de Woestijne)

Let $f(X) \in \mathbf{Q}[X]$ be a polynomial of degree n with rational coefficients. Suppose that degree(gcd$(f(X), f^{(k)}(X))) > 0$ for $k = 1 \ldots n-1$. Prove or disprove $f(X) = a(X-b)^n$, for some rational numbers a, b. (See also arXiv:math.AC/0605090, 3 May 2006.)

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