Problemen/UWC

Universitaire Wiskunde Competitie

Problem A (Proposed by Matthijs Coster)

Prove or disprove the following:
In a 9×9 Sudoku–square one randomly places the numbers $1 \ldots 8$. There is at least one field such that if any of the numbers $1 \ldots 9$ is placed there, the Sudoku–square can be filled in to a (not necessarily unique) complete solution.

Problem B (Proposed by Jaap Spies)

Imagine a flea circus consisting of n boxes in a row, numbered 1, 2,. . ., n. In each of the first m boxes there is one flea ($m \leq n$). Each flea can jump forward to boxes at a distance of at most $d = n - m$. For all fleas all d+1 jumps have the same probability.
The director of the circus has marked m boxes as special targets. On his sign all m fleas jump simultaneously (no collisions).

1. Calculate the probability that after the jump exactly m boxes are occupied.
2. Calculate the probability that all m marked boxes are occupied.

Problem C (Proposed by Klaas Pieter Hart)

We are given two measurable spaces $(X, \mathcal{A})$ and $(Y, \mathcal{B})$ plus a sub-σ-algebra $\mathcal{C}$ of $\mathcal{A}$. We are also given a real-valued function f on $X \times Y$ that is measurable with respect to the σ-algebra $\mathcal{A} \otimes \mathcal{B}$ generated by the family $\{A \times B : A \in \mathcal{A}, B \in \mathcal{B}\}$. Furthermore, each horizontal section f_y is measurable on X with respect to $\mathcal{C}$. Prove or disprove: f is measurable with respect to $\mathcal{C} \otimes \mathcal{B}$.

Problem * (Proposed by B. Sury)

Prove or disprove that if $\binom{2n+1}{n} \equiv 1 \bmod n^2 + n + 1$ where $n^2 + n + 1$ is a prime, then $n = 8$.

Eindredactie: Matthijs Coster
Redactieadres: UWC/NAW
Mathematisch Instituut
Postbus 9512
2300 RA Leiden
uwc@nieuwarchief.nl