Problemen/UWC

Universitaire Wiskunde Competitie

Problem A

In what follows f, g are two continuous functions.
1) Determine $f : \mathbf{R} \to \mathbf{R}$ and $g : \mathbf{R} \to \mathbf{R}$ such that $f \circ g(x) = x + 1$ and $g \circ f(x) = x - 1$.
2) Determine $f : \mathbf{R}^+ \to \mathbf{R}$ and $g : \mathbf{R} \to \mathbf{R}$ such that $f \circ g(x) = x + 1$ and $g \circ f(x) = 2x$.
As usual, the symbol '$\circ$' denotes the composition of functions and $\mathbf{R}^+$ the set of all strict positive real numbers.

Problem B

1. Let G be a group and suppose that the maps $f, g : G \to G$ with $f(x) = x^3$ and $g(x) = x^5$ are both homomorphisms. Show that G is Abelian.
2. In the previous exercise, by which pairs (m, n) can $(3, 5)$ be replaced if we still want to be able to prove that G is Abelian.

Problem C

For $s > 1$ define

$$\Phi_1(s) = \prod_p (1 - \frac{1}{p^s})^{-1} \text{ with } p \text{ over all primes } \equiv 1 \bmod 4$$

and

$$\Phi_3(s) = \prod_q (1 - \frac{1}{q^s})^{-1} \text{ with } q \text{ over all primes } \equiv 3 \bmod 4.$$

Describe how $\lim_{s \downarrow 1} \frac{\Phi_3(s)}{\Phi_1(s)}$ can be computed to 'any' degree of (high) accuracy (precision). (The use of an algebra-package is permitted.)

Star Problem

It is known that $\sum_{n=1}^{\infty} \binom{3n}{n} (\frac{1}{8})^n = \frac{3}{\sqrt{5}}$. This can be checked with Mathematica or Maple. Does there also exist an elementary proof for this equality?

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