Problemen/UWC

Universitaire Wiskunde Competitie

Problem A

A student association organises a large-scale dinner for 128 students. The chairs are numbered 1 through 128. The students are also assigned a number between 1 and 128. As the students come into the room one by one, they must sit at their assigned seat. However, 1 of the students is so drunk that he can't find his seat and takes an arbitrary one. Any sober student who comes in and finds his seat taken also takes an arbitrary one. The drunken student is one of the first 64 students. What is the probability that the last student gets to sit in the chair assigned to him?

Problem B

We consider amino acids A, B and C and proteins formed by an ordered sequence of these. We also consider 9 enzymes which modify the proteins by replacing two adjacent amino acids by two other amino acids. These substitutions are given by:

$$AA \to BC,\ \ AB \to CC,\ \ AC \to BA,$$
$$BA \to CB,\ \ BB \to CA,\ \ BC \to AA,$$
$$CA \to BB,\ \ CB \to AC,\ \ CC \to AB.$$

We define classes of proteins as follows. If a protein has been modified by an enzyme, then it still belongs to the same class of proteins. Two proteins belong to two different classes of proteins if there doesn't exist a set of enzymes that is able to modify one of the proteins into the other.

1. How many classes of proteins consisting of 12 amino acids do there exist,
2. How many proteins belong to the class of proteins of $ABCCBAABCCBA$?

Problem C

In what follows, P stands for the set consisting of all odd prime numbers; M is the set consisting of all natural 2-powers $1, 2, 4, 8, 16, 32, \ldots$; T is the set consisting of all positive integers that can be written as a sum of at least three consecutive natural numbers.

1. Show that the set theoretic union of P,M, and T coincides with the set consisting of all the natural numbers..
2. Show that the sets P,M, and T are pairwise disjoint.
3. Given $b \in T$, determine $t(b)$ in terms of the prime decomposition of b, where by definition $t(b)$ stands for the minimum of all those numbers $t > 2$ for which b admits an expression as sum of t consecutive natural numbers.
4. Consider the cardinality $C(b)$ of the set of all odd positive divisors of some element b of T. Now think of expressing this b in all possible ways as a sum of at least three consecutive natural numbers. Suppose this can be done in $S(b)$ ways. Determine the numerical connection between the numbers $C(b)$ and $S(b)$.

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