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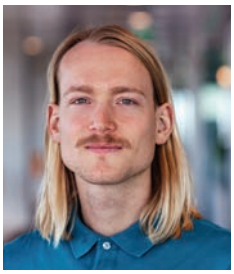

Eye on Variety – The teacher's work in developing mathematical whole-class discussions
Chris Kooloos

In May 2022, Chris Kooloos from Radboud University Nijmegen successfully defended his PhD thesis with the title *Eye on Variety – The teacher's work in developing mathematical whole-class discussions*. Chris carried out his research under the supervision of prof. dr. Gert J. Heckman (Radboud University), prof. dr. Rainier H. Kaenders (University of Bonn) and dr. Helma W. Oolbekkink-Marchand (Radboud University & HAN University of Applied Sciences).

Since 2021, Chris has been working as 'vakdidacticus' mathematics at Radboud University. In this position, he develops and provides master courses that are especially interesting for educational master students.

Teaching students to think mathematically

In the last century much has changed regarding mathematics education, attention has shifted from teaching mathematics as a set of rules and procedures toward teaching for mathematical thinking. The mathematical content should be supportive in — as well as resulting from this general analytical way of thinking. In the end, mathematics is at first a human activity, and not only a collection of procedures in which things need to be done in a certain way.

Research into student-centered mathematics education has flourished and provided many insights, models, and tools regarding teaching for mathematical thinking. In his thesis, Chris considered one specific teacher practice with regard to developing students' mathematical thinking: discourse-based teaching, or, in other words, teaching through whole-class discourse that builds on students' various ways of thinking. Discourse-based teaching relies on teaching mathematics lessons by

1. deciding what mathematics students should learn;
2. thinking about problems or questions that, if students work on them and discuss their ideas, can lead the students to learning to deal with that particular mathematics;
3. encouraging students to work on the problems and questions;
4. orchestrating a whole-class discussion about students' various ideas and solution methods.

During his PhD, Chris collaborated with a group of Dutch mathematics teachers and supported them in developing mathematics lessons based on whole-class discussions. They all had a common vision. Namely, to create opportunities for their students to experience variation in mathematics, create a learning environment in class where students can discuss their mathematical ideas freely, experience that mathematics can be done in different ways and is something they already do, and to realize that by discussing

Pas gepromoveerden brengen hun werk onder de aandacht. Heeft u tips voor deze rubriek of bent u zelf pas gepromoveerd? Laat het weten aan onze redacteur.

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their thinking they collectively learn new mathematics! A very novel vision to strive for.

Discussing to foster understanding

For teachers, discourse-based teaching is a valuable practice to gain insight into students' thinking, to build the lesson on their thinking, and to guide them toward learning important mathematical ideas. In discourse-based mathematics lessons, students can also develop a corresponding disposition: that their thinking matters, and that through thinking and discussing their thoughts, they can discover important mathematical ideas. However, developing, and orchestrating classroom discourse about students' different solution methods is a complex task for mathematics teachers. Work, time, creativity, patience, and persistence are needed.

What makes the orchestration of productive classroom discourse particularly complex is finding a balance between building on students' ideas and guiding the students toward domain-specific goals, such as 'understanding' of mathematical concepts like tangent lines. Mathematical goals can only be achieved by building on student thinking and by supporting them in making the necessary steps they need toward understanding the topic. The teacher has the responsibility to make decisions about the ideas students share and to advance the mathematical learning of the whole group toward the disciplinary mathematical ideas. For classroom discourse to be productive, students sharing and discussing ideas is not sufficient. Some students' mathematical ideas are more advanced than others, some explanations are generalizable, and some are not. Here lies an essential yet challenging task for the teacher, namely orchestrating classroom discourse such that the students are both supported in making important mathematical connections, and are also guided toward disciplinary ideas. At the same time the focus should be maintained on students' ideas and reasoning.

A large part of Chris's research is devoted to professional development and teacher learning. He collaborated closely with a team of mathematics teachers who wanted to implement discourse-based teaching in their classrooms. Teachers were stimulated to develop and implement discourse-based teaching while staying close to their practice and had regular meetings to discuss and evaluate the progress. So, the work is very close to high school teaching practice. During this collaboration, Chris investigated how teachers learned and applied discourse-based teaching in their classrooms, how they reacted to the students' ideas, and how they tried to make sense of the students' mathematical learning.

In four lessons

Let us have a peak in one of these collaborations. During his research, Chris collaborated closely with tenth-grade teacher Anna in developing four discourse-based lessons in analytic geometry. The four lessons in analytic geometry consisted of students working on a mathematical problem plus classroom discourse concerning students' different solution methods. During the first lesson, the students were asked to think of the following problem:

Calculate the distance from point $P(6,1)$ to line $l: y = \frac{1}{3} + 4$.

The current presentation of analytic geometry in Dutch textbooks is very procedural and often consists of step-by-step instructions. For example, to calculate the distance from a point to a line, only one solution method is given. Generally, students are accustomed to memorizing and practicing such step-by-step procedures. During these lessons, instead of providing students with a single procedure, Anna presented them with open problems, and orchestrated classroom discourse about students' different solution methods. When students were asked to calculate the distance from a point to a line, several possible solution methods were formulated and discussed.

Moreover, by comparing various solution methods, students were supported in making important mathematical connections between different representations. Effective orchestration of classroom discourse shifts students' cognitive attention from problem solutions and procedural rules to sense-making and the reasoning that leads to a solution. In other words, instead of trying to get the right answer, mathematical discourse is about trying to understand and question each other's ideas and reasoning, and collaboratively deciding what is true based on logical argumentation.

From the analysis of the lesson observations, it is observed that the number of students who contributed to classroom discourse increased during the course of the four lessons. In the fourth lesson, 18 out of 23 students contributed to the discourse. In the first lesson, Anna talked twice as much as the students; in the second lesson, the students talked twice as much as Anna; and in the third and fourth lessons, the students talked slightly more than her.

A framework for teacher actions

During whole-class discourse, teachers should take many decisions on the spot, they need to build upon students' thinking and at the same time keep an 'eye on the mathematical horizon'. Chris constructed a theoretical framework to characterize the teacher's actions in classroom discourse, to describe the change in the teacher's role in learning discourse-based mathematical teaching, and to describe how the teachers tried to make sense of the mathematical thinking of the students.

The teacher's actions — utterances — in interaction during whole-class discourse are crucial in supporting the students to express their mathematical thinking, engage with each other's thinking, and make important mathematical connections. In a way, the teacher's actions are the gateway between everything the teacher knows, thinks, wants, and feels on the one hand and the students on the other hand. A teacher can choose to give a hint or to ask another student to think of the next step in trying to solve a problem. The concrete actions the teachers choose to coordinate whole-class discourse affect the learning process of the whole class.

A theoretical framework for the teacher's actions can give concrete characteristics of the attitude and work of teachers during classroom discourse. In the framework Chris developed teacher actions are divided into four categories. Those are: *convergent actions* (that help converge to a solution) such as demonstration and reformulation, *divergent actions* (that stimulate interaction and discussion between students) such as requesting explanations and clarifications, *encouraging actions* such as confirmation and encouragement, and *regulatory actions*, to make sure that the discussions are regulated. A combination of teacher actions can guarantee that there is a balance between building on students' ideas and guiding the students toward disciplinary ideas.

During the four lessons on analytic geometry, Chris made video recordings to observe how Anna orchestrated the discussions. Analysis of video recordings from the lectures revealed three main

changes in Anna's role. First, the way she reacted to correct or incorrect solution methods shifted from confirming or setting aside suggestions, toward making the solution methods the subject of discussion. Second, the distribution of turns changed such that more students were involved in the discourse and in reacting to each other's solution methods. Third, her actions shifted from convergent, teacher-led actions toward divergent, student-led actions. These results show that within four lessons, an important step has been taken toward establishing a discourse community.

Thinking like the student

During his research, Chris also explored the decision-making patterns of five experienced Dutch mathematics teachers during their novice attempts at orchestrating whole-class discourse. His goal was to understand how teachers try to make sense of the students' ideas and thoughts. During whole-class discussions teachers need to adapt to the perspective of the students, they need to imagine what the students' line of thinking could be, so they can help each student build upon, or change something in their line of thinking. How teachers understand the thoughts of the students influences their actions during whole-class discussions. The mathematical experience of teachers plays an important role in this process. Using video recordings of the lessons they reflected upon and discussed the whole-class discussions, and how the teachers and the students reacted to certain situations.

Chris grouped the observed teacher sense-making-actions in five categories concerning the role of teacher's mathematical thinking in their sense-making: *flexibility*, *preoccupation*, *incomprehension*, *exemplification*, *and projection*. Flexibility refers to the ability of a teacher's mathematical thinking to discuss openly various perspectives of a problem and to consider different solution methods proposed by students. On the other hand, preoccupation refers to the situation the content of teachers' mathematical thinking is limited to one solution method or a particular expected answer, this inflexibility can impede them in adopting a student's perspective that doesn't match their own thinking.

Incomprehension describes the situation a teacher does not mathematically understand a student's solution method or idea, this can impede sense-making: if the incomprehension does not get resolved, the teacher is not able to fully investigate the student's meaning and reasoning. However, such incomprehension can also foster adopting the perspective of other students in the classroom. The process that the teachers' mathematical thinking can serve as an example of the mathematical thinking they would like to develop in their students is described as exemplification. It can support teachers in articulating what kind of thinking they aim to develop in their students as well as in recognizing to what extent students already articulate such thinking. Finally, projection refers to teachers projecting their own ways of mathematical thinking onto their students, without considering the differences between their own expert thinking and the thinking of learners, this can impede them in adopting the students' perspective.

These categories show how both the content and the process of teacher mathematical thinking can support or impede students' sense-making. Chris's findings suggest that sense-making of students' mathematical thinking requires teachers to (re-)engage in reflective thinking regarding the mathematical content as well as the process of their own mathematical thinking.

The more personal aspect

As a final note we would like to give the word to the doctor.

Chris, how did you get interested in mathematics education?
"During my mathematics studies, I regularly provided tutoring and exam training for high school students and tutorials at the faculty, and becoming a teacher was sort of a plan B for me. When I actively started to orient myself on career options during my research master, I realized I would prefer to stand in front of young people all day over sitting behind a computer. One month after I had started teaching, I spoke to Wim Veldman, who had supervised my master thesis, and he told me there would be a PhD position in mathematics education. I had never realized that was something one could do a PhD in! For several years, I combined my teaching job with PhD research in mathematics education, and gradually became more and more aware and sure that the learning and teaching of mathematics would be the focus of my career."

Some other thoughts you would like to share with the readers?
"My work with a group of mathematics teachers was very important for my PhD project and opened my eyes to all the interesting mathematics that is involved in high-school mathematics, once people really start to think and talk about it. I have learned a lot of mathematics from these teachers and their students. Furthermore, I would say that this kind of teaching can also help younger students and their teachers, as well as higher mathematics education. For example, if university mathematics students would be supported (more) in thinking mathematically and talking about their thinking, I believe more students would succeed in finishing their studies (and enjoying mathematics!).

Mathematics Education faces many challenges. The most important is probably the huge shortage of mathematics teachers. In my position, I make an effort to recruit potential teachers by showing mathematics students how teaching mathematics involves so much more than being able to give good explanations, and that as a teacher, you learn a lot of mathematics, if you encourage students to think and talk and if you listen to what they say. It would be great if we, as mathematicians, would all spread the message that teaching mathematics is a very respectable and mathematical profession. Another important challenge is teaching students of all ages mathematics in such a way that they become aware of and develop their mathematical thinking and see the use and beauty in mathematics. I hope to continue cooperating with researchers, teachers, and students at multiple levels of mathematics education in making a modest contribution to facing both challenges."

Concluding

To summarize, in his research Chris investigated how teachers can organize their mathematics teaching using whole-class discourse. He collaborated closely with a group of mathematics teachers and supported them in developing their lessons. Teachers were stimulated to develop and implement discourse-based teaching while staying close to their practice. They met regularly to discuss and reflect upon the lesson recordings, and they tried to unravel the complex process of how teachers try to make sense of the thoughts and ideas of the students. It is beautiful to see such collaborations flourishing, we hope to see similar projects in the future! This project shows what is possible when enthusiastic people join their forces. We wish Chris the best with his future work!