This Problem Section is open to everyone; everybody is encouraged to send in solutions and propose problems. Group contributions are welcome. We will select the most elegant solutions for publication. For this, solutions should be received before 15 October 2021.
The solutions of the problems in this issue will appear in one of the subsequent issues.

Problem A (proposed by Daan van Gent)
Write $T = (\mathbb{Z}/8\mathbb{Z})^2$ for the torus chessboard. For every square $t \in T$ its neighbours are the squares in the set $\{t + d \mid d \in D\}$ for $D = \{-1, 0, 1\}^2 \setminus \{(0,0)\}$. A line is a set of squares of the form $\{t + nd \mid n \in \mathbb{Z}\}$ for $t \in T$ and $d \in D$. The following figure gives an example of the neighbours (coloured dark grey) of the dotted square, as well as an example of a line (coloured light grey). Disprove or give an example: There exists a pairing of neighbouring squares, i.e. a partitioning of $T$ into sets $\{s, t\}$ of size 2 such that $s$ and $t$ are neighbours, such that every line contains a pair.

Problem B (proposed by Hendrik Lenstra)
Write $\varphi$ for the Euler totient function. Determine all infinite sequences $(a_n)_{n \in \mathbb{Z}_+}$ of positive integers satisfying $\varphi(a_{n+1}) = a_n$ for all $n \geq 0$.

Problem C (proposed by Hendrik Lenstra)
Let $R$ be a ring. We say $x \in R$ is a unit if there exists some $y \in R$ such that $xy = yx = 1$, and $x \in R$ is idempotent if $x^2 = x$. Show that if every unit of $R$ is central, then every idempotent of $R$ is central.