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History

Willem Jacob ’s Gravesande and mathematics in the early eighteenth century

What did mathematics look like in the early eighteenth century and what did it mean to be a mathematician at the time? Questions such as these are central to understanding the historical development of mathematics more generally but are notoriously hard to answer. In this article Jip van Besouw attempts to take some small steps towards an interpretation. He will do so by looking at one particular mathematician, Willem Jacob ’s Gravesande (1688–1742). Although ’s Gravesande is not one of the household names in the history of mathematics, he was located at a central node of the mathematical community during his era. For most of his career he held a prestigious chair of mathematics and astronomy at Leiden University, one of Europe’s leading institutes of learning.

Mathematics as a calling?

Among historians, ’s Gravesande is known as a key figure in the physics and natural philosophy of the early eighteenth century. An outspoken supporter of the work of Isaac Newton (1642–1727) and an inspiring teacher, ’s Gravesande is usually portrayed as a powerful proponent of the new experimental-mathematical physics. By comparison, little attention has been paid to his mathematical work. Much of this has to do with the fact that ’s Gravesande made few original mathematical contributions. Moreover, the contributions he actually made were either in fields that are now deemed peripheral — such as perspective geometry — or were quickly superseded by more advanced works of giants such as Johann Bernoulli (1667–1748) and Leonard Euler (1707–1783). If, however, our aim is to understand what mathematics was like in practice, as a human endeavour, there are very good reasons for taking a second look at ’s Gravesande.

In important ways ’s Gravesande’s career as a mathematician was typical for the era in which he was living. One of the typical features was his route to professional mathematics. Rather than via formal training or mentorship, ’s Gravesande was largely self-trained and his mathematical activities were avocational. At the same time his interests in mathematics were fostered by close relatives: both his father and his maternal grandfather had a keen interest in practical mathematics. ’s Gravesande was born into one of the leading families in the governance of the city of ’s-Hertogenbosch. His father Dirk (1646–1716), while being a magistrate of that city, communicated with his luminary compatriot Christiaan Huygens (1629–1695) on the grinding of lenses and applied for at least one patent, on the construction of horizontal windmills. Our ’s Gravesande’s grandfather, Nicolaas Blom (d. 1693?), was an overseer of fortifications and a surveyor and as such almost certainly steeped in geometry [3, pp. 233–235].

Because of this we can expect that the youthful ’s Gravesande found ample opportunity and resources to develop his mathematical talents. When he moved to Leiden to attend university there between 1704 and 1707 — graduating as a teenager — he however chose to study law. According to legend he did so to please his father and would actually have spent his time in classes to work on what would become his first book, the *Essai de perspective*, published in 1711 [1, Vol. 1, p. xi, note F]. It is quite unclear whether ’s Gravesande attended any courses in mathematics at Leiden. What would not have helped was that mathematics was at a low point at the university. The chair of mathematics was held by Burchard de Volder (1643–1709) but De Volder stopped his teaching activities in 1705 due to illnesses. Moreover, he preferred natural philosophy over mathematics. No substantial teaching of mathematics seemed to have taken place between 1705 and 1717, when ’s Gravesande himself assumed the chair. There was a lecturer in military mathematics and surveying, Henri Coets (d. 1730), a known expert on Euclid’s *Elements*. However, his lectures probably included the same kind of geometry ’s Gravesande had seen at home [5, pp. 24–25].

Where then, if not at the university, could a person like ’s Gravesande sharpen his mathematical skills? Part of the answer is through contacts and correspondence with others engaged with mathematics, often at leisure. After graduating ’s Gravesande moved to The Hague, where he apparently practiced as a lawyer but also found his way into the world of intellectual sociability known as the Republic of Letters. An important contact was the Brit William Burnet (1687–1729), son of the Bishop of Salisbury. Burnet was a member of the Royal Society of London, a protégé

Figure 1 Portrait of 's Gravesande by Jacob Houbraken.

of Isaac Newton himself, and a resident of The Hague, too. It was this Burnet, who would later go on to become the governor of New York and various other British American colonies, who brought 's Gravesande to the attention of Johann Bernoulli and, very likely, to Newton's as well. Burnet wrote to Bernoulli in 1710 that if Bernoulli wanted to reach him he could send his letters to "Mr. De Sgravesande" who "has the most original genius for mathematics, but, as he is a lawyer, cannot give himself to it entirely".[1]

Through such contacts and introductions 's Gravesande gradually came in touch with other mathematicians in the Republic of Letters. Quite soon after this we find him corresponding with Nicolaus Bernoulli (1687–1759), one of Johann's nephews. In this correspondence, to which I will get back below, they discussed among other things logarithms and a particular problem in statistics. It was for an important part through epistolary interactions such as these that the new mathematics of the age was developed, whether it was calculus or statistics. 's Gravesande was very active in these matters. In 1713 he became one of the founding editors of the *Journal litéraire*. The main business of this *Journal* was to produce book reviews of any kind of book, with subject matters ranging from theology and modern literature to mathematics and natural philosophy. 's Gravesande used this outlet to publish some of his own tracts but the journal also published several polemical pieces on the dispute over the priority over the calculus between Newton and Leibniz. These were written by close allies of the two antagonists such as John Keill (1671–1721) and Jacob Hermann (1678–1733) [14, p. 211], and very likely solicited by 's Gravesande.

's Gravesande applied his talents for networking also in other areas, namely to befriend people with political power. In his first direct contact with Johann Bernoulli, 's Gravesande wrote in the name of one particular governor of Leiden University, a former general of the Dutch army, who was attempting to hire Bernoulli for the chair of mathematics (Bernoulli deemed the offer financially inadequate). 's Gravesande generally seems to have been happy to employ his political connections for mathematical purposes and vice versa. For example, 's Gravesande knew several of the top diplomats of the Netherlands. He dedicated his first book, the *Essais de perspective*, to one such diplomat and designed a sundial for another. This second diplomat, Arent Wassenaer van Duyvenvoorde (1669–1721), hired 's Gravesande a couple of years later as a secretary on a diplomatic trip to London (1715–1716). 's Gravesande used this trip to get into contact with Newton and various other members of the Royal Society. He was promptly made a Fellow of that society, from which we can infer that he made a strong impression on the British mathematicians. When Leiden University decided to offer 's Gravesande a chair in mathematics and astronomy in 1717 it was in turn on the formal recommendation of both Duyvenvoorde and Isaac Newton [3, p. 240].

’s Gravesande’s route towards professional mathematics, to sum up, started with the opportunities and knowledge offered by his family but was paved by the reputation he built in the Republic of Letters. Of particular importance was the fact that he was well regarded in that Republic’s tiny province of advanced mathematics. To get entrance to that province and be able to communicate on equal footing with the likes of the Bernoullis and confidants of Newton was exceptional in itself. The fact that he got the chair of mathematics and astronomy in Leiden, however, was in no small way a consequence of the backing he received from his contacts in the circles of diplomacy and higher politics. These were clearly of great help in securing a living as a mathematician.

Natural philosophy and mixed mathematics

When ’s Gravesande managed to secure a mathematical vocation, we would expect him to start ramping up on a serious mathematical project. Both statistics and infinitesimal calculus were budding fields in 1717 and faced many opportunities and challenges. Quite unexpectedly, at least from a modern point of view, is that ’s Gravesande chose to turn towards what we would now consider experimental physics. This move, too, can tell us something about the field of mathematics at the time. Although ’s Gravesande’s choice was certainly guided by his personal taste it is important to realise that the relation between mathematics and physics has evolved much over time. Both were very different disciplines and genres than they are nowadays.

To begin with we should look at what was expected from a mathematics professor. University professors were appointed to lecture, and, explicitly so in Leiden, to serve the interest of their country. Much of a professor’s workload was shaped by educational demands. Although original contributions to their fields were appreciated these were not supposed to be their main affair. What is more, university mathematics had a mere propaedeutic function. The regular university curriculum of the time offered only elementary mathematics and one could not graduate in it. This programme was adjusted to the needs and capabilities of the incoming students. Few of them would have been able to follow anything at all of the newer mathematics. As was the case for ’s Gravesande himself, students were often in their mid teens when they joined Leiden University and had generally attended so-called Latin Schools before. Once they arrived at Leiden students graduated mostly in law — the degree of choice of the ruling elite — or medicine. They were supposed to go on to administrative and medical careers respectively [15, Chapter 1]. That ’s Gravesande taught only elementary mathematics and physics was therefore both desirable for the university and suited to the background of his students.

None of this takes anything away from the fact that ’s Gravesande could of course have spent his spare time on mathematical problems. Both of the Bernoullis with whom he corresponded, Johann and Nicolaus, combined their chairs of mathematics with strings of top notch mathematical papers. ’s Gravesande worked on topics in mathematics proper only sporadically. Most of his advanced work went into designing instruments and machines and coming up with new experimental setups to test physical theories and establish physical phenomena. Can we therefore say that he turned his back on mathematics in favour of physics?

’s Gravesande himself took a different perspective. In his bestselling *The mathematical basics of physics, confirmed by experiment; or, an introduction to Newton’s philosophy*, first published in 1719, he distinguished between ‘mixed’ and ‘pure’ mathematics and claimed that “physics belongs to mixed mathematics” [10, Vol. 1, ii]. By his definition, doing physics was doing mathematics as well. It is important to realise here that mixed mathematics was not simply what we now call applied mathematics [13]. It referred instead to sciences we would now qualify as physical, such as mechanics, optics, and hydrostatics, as far as they were treated quantitatively. The refraction of light through lenses was a typical example of an optical subject which one would attack mathematically, in this case by advanced geometry. The natures of light and colour, on the other hand, were generally considered to be topics for qualitative natural philosophising and where therefore outside of the domain of mathematics in the seventeenth century.

What was it that classified sciences such as mechanics and optics as mathematics for ’s Gravesande? Above all it was the fact that physical sciences were supposed to deal with natural motions. According to a view that is often called ‘corpuscularism’, natural phenomena were to be explained in terms of the motions of material bodies or particles. In one way or another most of the major natural philosophers of the seventeenth and eighteenth centuries adhered to this view. As ’s Gravesande explained himself: “All things in physics are accomplished by motion; because no change can be made to bodies […] except that which is made by motion.” He continued to argue that “Motion itself is a quantity; it can be increased and diminished; whatever therefore attends to [motion], that is, all in physics, ought to be treated mathematically.” The centrality of this point of view cannot be overestimated. What ’s Gravesande believed, like many of his contemporaries, was that motion is a quantity. Mathematical physics was not just mathematics applied to physics: physical motion was a mathematical topic by its very essence [7, p. 15].

In the practice of his physics ’s Gravesande used mathematical approaches in various ways. These included expressing physical concepts quantitatively and measuring these concepts through tightly controlled experiments. The mathematics involved generally did not exceed solving the equivalent of quadratic equations or approximating the numerical values of square roots. Of interest from a mathematical side is the use of advanced curves to describe particular motions. In his *The mathematical basics of physics*, ’s Gravesande used for example sinusoids to find the dimensions of the rainbow, cycloids to describe centre of oscillation of pendulum motion, elliptical and near circular motions when discussing the more intricate parts of Newton’s celestial mechanics, and logarithmic curves in his quest to describe the deceleration of bodies moving in a fluid. It is unclear to what extent ’s Gravesande presented any significant new results in doing so, whether mathematical or physical. Potential innovations are obfuscated by his tendencies not to refer to other authors and to abridge the solutions to mathematical problems [4, pp. 27–35]. These tendencies, consequences of pedagogical choices, necessitate the historian willing to find what is new to study the mathematical demonstrations in ’s Gravesande’s book in detail. For the moment, this remains an opportunity for future historians of mathematics or perhaps students looking for an interesting topic for a bachelor or master thesis.

The kind of geometry used by ’s Gravesande perfectly illustrates the overlaps and the divides between mathematics and physics around 1700. Although many of the physicists of the time were interested in mathematical curves, we should keep in mind that only a dozen or so of the best geometers were able to publish new results on a regular basis. For some of them, physical problems were predominantly a way to generate new curves and therefore new mathematical objects to work with. Johann Bernoulli, for example, often cared more about the manipulation of these mathematical objects than about the physical problems they presented. For Newton and ’s Gravesande the opposite was true. In their hands, physics was redefined not just a branch of mixed mathematics, it was actually considered as the most important part of mathematics per se. As we will see in the next section ’s Gravesande did not just state that physics was a game to be played by mathematicians: it was the game all mathematicians should play.

The pull of practical mathematics

One of ’s Gravesande’s first outings as a mathematician steers us directly towards his reasons for arguing so. In a letter addressed to “Monsieur B*** de la Société Royale de Londres”, presumably Burnet, whom he gave permission to publish it, ’s Gravesande reacted to a review of a book on the new analysis of integrals and differentials. This book had been written by Père Reynaud (1656–1728), one of the many French mathematicians involved in the early development of the calculus, and was reviewed in 1709 by Jean le Clerc (1657–1736), a Dutch theologian and man of letters. Le Clerc, not a mathematician himself, had argued in his review that the usefulness of the great abstractions of mathematics, such as could be found in Reynaud’s book, lied only in their capacity to amuse the mind of man.

’s Gravesande’s exasperated letter consists for a large part of counterarguments to this claim, in particular via a summation of human activities made easier by mathematics. These included timekeeping; “the art of throwing bombs”; the calculation of the dimensions of large telescopic lenses, made easier particularly by the new analysis in question; and navigation. ’s Gravesande furthermore added that “multiple other parts of mathematics” such as surveying, the study of sundials, and so on, were of great use to life. Thus we see him implying again that such practical concerns are an integral part of mathematics. What is striking about the letter is that ’s Gravesande seems to argue that the usefulness of mathematics lies completely in its role in improving technology. This is implied in his statement that, if more people would be convinced of the usefulness of mathematics, “we would not see so many fruitless machines” made by “ignorant inventors”.[2]

This same theme can be found in ’s Gravesande’s inaugural oration in Leiden, held in 1717. ’s Gravesande there repeated that mathematics was of great help in navigation, in water management, and in timekeeping. He now singled out astronomy, as set out in Newton’s *Principia*, as being of great utility to these practicalities. However, he also introduced a second way mathematics could be useful in this text. This was in studying ‘the art of reasoning’. Mathematical reasoning, ’s Gravesande explained, dealt with quantities, which were simple and abstract ‘ideas’, and was built upon a axiomatic-deductive method, the most reliable method available. Since any form of reasoning depended on the comparison of ideas, the basics of reasoning could best be learned from starting from the simplest ideas. Hence mathematics was a very suitable way to learn how to reason logically, a skill that could afterwards be applied to other fields [7], see also [4]. This was of course in perfect agreement with the propaedeutic function mathematics had in the university’s curriculum at the time and ’s Gravesande was at least partly preaching to the choir.

What is conspicuously absent in both of the two texts considered here is the idea that doing mathematics has value in itself. In later texts ’s Gravesande indeed made it clear that he did not believe that there was such an intrinsic value. In 1727 he published a book on elementary mathematics for his courses in Leiden and argued on the very first pages that one should always keep one eye on possible applications. For if one would only study abstract mathematics, one “does not learn the proper mathematical method, but creates a disposition suitable only for reasonings about quantity”, he claimed [8, Praefatio]. According to his biographer ’s Gravesande “looked down upon those professional calculators spending their lives on the study of purely speculative truths of which the discovery is of no use, neither with regards to other sciences, nor with regards to the needs of life” [2, Vol. 1, xxiii].

Perhaps dismaying to modern mathematicians, ’s Gravesande thus located the usefulness of practicing mathematics exclusively in applying the knowledge gained elsewhere. Ironically, when ’s Gravesande was at some point asked to teach courses in practical mathematical topics such as surveying, he expressed his relief when the university later hired an additional lecturer to replace him for these somewhat less-than-academic subjects [9], see also [17, pp. 31–32]. Still, almost all of his contributions to mathematics proper were made in contexts of practical applications. It is to these contributions that we will turn in the final sections.[3]

Perspective problems

Although there exists scholarship on some of ’s Gravesande’s mathematical pieces, ’s Gravesande has usually not been the primary focus of these studies. Historical work remains to be done if we are to see the merit of ’s Gravesande as a mathematician. This work is made more complex by his own attitude towards the subject: exactly because he did not deem it important in its own right, he never drew attention to the innovations he made mathematically but only to the application of his work.

This is the case as well in his *Essai de perspective*, his first real scholarly work published in 1711. The book was praised in an anonymous review, perhaps by Johann Bernoulli, who at least wrote in a personal communication to ’s Gravesande that he “wished that you take the trouble to write on the other parts of optics with the same clarity and the same skill as you have done on perspective”. ’s Gravesande’s friend Burnet was similarly impressed. In the letter to Bernoulli quoted above, Burnet said of ’s Gravesande that “he has made beautiful discoveries in Perspective, whereof he will soon publish something, which I will send you right away” [3, p. 238]. These discoveries, however, were not at all highlighted by the author himself. In the preface ’s Gravesande asserted that his main goal was to be of use to painters who wished to use the theory of perspective, and that therefore he had tried to find a middle ground between the mathematical theory and its application.

The book itself consists for a large part out of problems one would face when trying to draw some scene in perspective. Particularly, 's Gravesande gave the solution for problems of the form: construct the image of a certain point or object, given a certain configuration of the image plane and the object. One of 's Gravesande's aims was surely to keep the mathematics at an elementary level. In the preface he spent some time to explain that he had published the easiest solutions to particular problems and in many cases multiple solutions so that the reader could choose according to particular preferences. In various cases 's Gravesande gave a solution that could be carried out without the use of even a compass [6, e.g. p. 39 and p. 43] As he explained in the preface, almost all of the solutions were carried out with what one could learn from Euclid's *Elements*: where this was not the case the text was set in italics so as to warn the reader.

's Gravesande's treatise on perspective is his only mathematical publication that has been analysed at some length by a modern historian. In her history on perspective geometry, Kirsti Andersen has dedicated one chapter to 's Gravesande, discussing in particular how his methods relate to those of his predecessors [2]. Andersen also has drawn attention to some of the more difficult problems solved by 's Gravesande. One of those is to determine what parts of a curved column base such as that shown in Figure 2 would be visible from a given eye point.

's Gravesande considered the base of the column as consisting of horizontal discs of different radii laying on top of each other, with parts of each disc being hidden from view by the overlaying disc. Consider for example the circle $DFEL$ of 'Fig. 34' of Figure 2, which represents a horizontal cross-section of the column base. Draw in this plane the projection $CDHE$ (from the eye point O) of the disc above it. The points D and E mark the boundary between the visible and the obstructed parts of the circumference of the circle. So determining these points is the first step to drawing a faithful perspective view of the column.

's Gravesande found that the distance AG from the centre of the smaller circle A

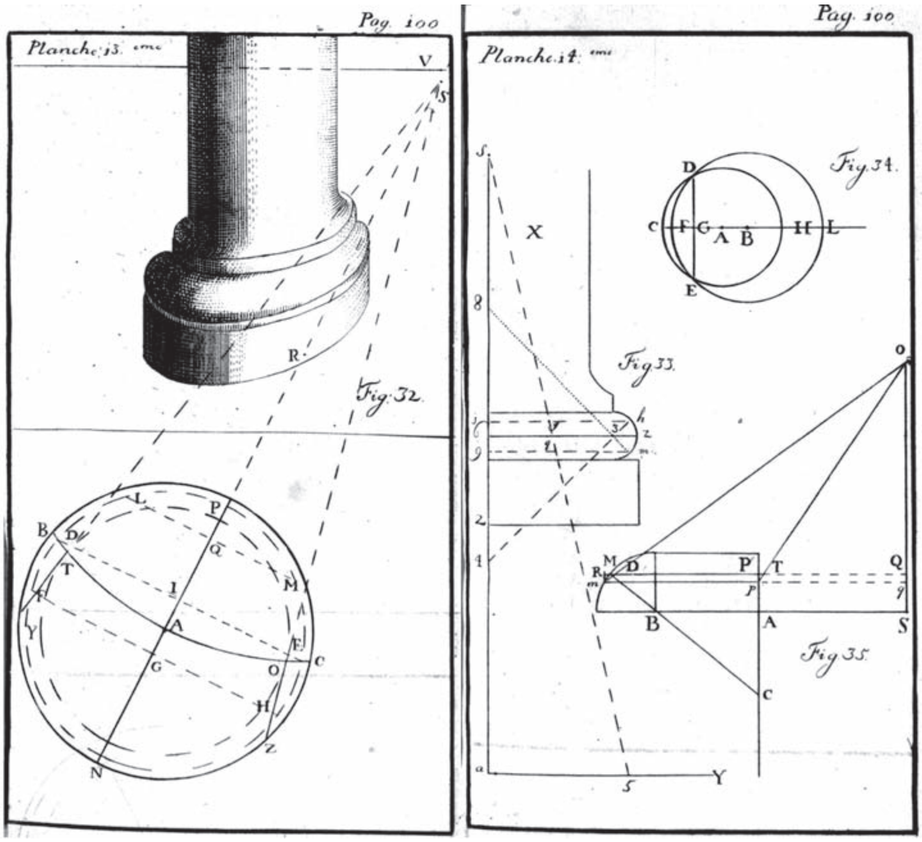


Figure 2 's Gravesande's analysis of the perspective view of the curved pedestal of a column.

to the chord DE was $\frac{b^2-a^2}{2c}-\frac{c}{2}$, where a and b are the radii of the two circles, and c the distance between their centres, after the projection. In order to account for a continuously curved shape such as the column base shown in the figure, 's Gravesande introduced Leibnizian infinitesimals to deal with the limit case where the distance between the discs becomes vanishingly small. He found that, in this case, AG becomes $\frac{xydy}{edx}-\frac{y^2}{e}$, where $y(x)$ is the shape of the column profile in the coordinate system defined by $OQ=x$ and $MP=y$, and $e=AS$ in 'Fig. 35'. This expression can be geometrically interpreted and constructed (in particular, $\frac{ydy}{dx}$ is the subnormal PC), and hence the visibility points D and E for a given cross-section of the column base can be obtained [2, pp. 735–738; 6, pp. 89–100].

What is interesting here is that 's Gravesande, who was still an unknown figure in the mathematical world when he wrote this, used the new infinitesimal calculus to attack a kind of problem to which it had not been applied before. As he makes clear in a note at the end of the solution he could have done it "without algebra", that is, strictly geometrically, but used the infinitesimals because it made the proof much shorter. Apparently, 's Gravesande was not averse of taking on particularly difficult problems and reducing them to simpler problems by whatever means he found to be working.

Binomial number crunching

A second mathematical topic 's Gravesande was involved in was statistics. One particular occasion to arouse his interest was a paper presented to the Royal Society in 1710 by John Arbuthnot (1667–1735), which is a very early example of statistical hypothesis testing in social science. The paper presents a table containing the number of baptised children in London for the previous 82 years. One seemingly spectacular feature of this data was that in each of these 82 years the number of boys was higher than that of the girls. Rather than figuring out what happened to those poor girl babies who were never baptised, the mathematicians dealing with it took the numbers as a proxy for the birth ratios and argued that the asymmetry was a sign of God's providence. Surely, God wanted every human being to have a life partner and had increased the number of baby boys because men were more likely to be killed than women due to their hunting and warring?

Leaving aside questions of infanticide and discrimination, what is of interest from a mathematical point in this episode is the handling of the notion of 'chance'. Throughout most of the discussion it was taken for granted that, if sex were determined by 'chance' rather than by providence, the probability for a newborn to be a boy was the same as for it to be a girl. Implicitly assuming this, Arbuthnot had argued that, if 'chance' were at play, the probability of there being more boys than girls was $\frac{1}{2}$ for each year — ignoring the particular case of the numbers being exactly equal. Arbuthnot treated the problem as throwing 82 'two-sided dies' and argued the probability of the London numbers coming up to have been $(\frac{1}{2})^{82}$.

Perhaps exactly because Arbuthnot's probabilistic treatment was quite crude, it inspired a number of other mathematicians to expand on it. 's Gravesande seems to have been the first of those. In a treatise written originally in Dutch and circulating among his contacts, he set out to give Arbuthnot's proof for divine providence "a new degree of force" and approached the problem as follows. Rather than concentrating on the feature of 82 boy years in a row, he tried to take into account that the sex ratio always fell within certain margins, with the proportion of boys being roughly between 0.503 and 0.536. His ultimate aim was to "determine just how much one should have bet" against the London numbers being between those margins given the supposition that "the birth of children is the effect of chance [*hazard*]" and to show that the number was much smaller than Arbuthnot's. As far as the ultimate conclusion is concerned, there does not seem to be much here. On Arbuthnot's calculation the bet was $1:4.836 \cdot 10^{24}$, which would surely already have convinced anyone willing to buy into the premises of the argument.[4]

In any case, 's Gravesande felt it was worth expanding on. The mathematical part of his treatise starts out with an elementary exposition of binomial expansion including a long corollary on "examining the formation of the coefficients obtained in bringing up a binomial to any power whatsoever". It does not seem that there was anything new in his treatment as the binomial coefficients he derived were widely known due to Pascal's work from the 1660's. Since 's Gravesande treated a single birth as a coin toss, with equal probability of a boy and a girl, binomials provided the appropriate method to attack the problem. His problem can be summarised as follows: what is the probability that, throwing a coin for every individual birth, the number of heads will be within the margins given above, not for a single series of tosses, but for 82 series in a row?

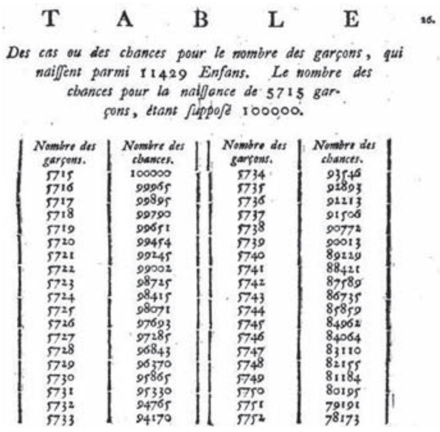
's Gravesande's approach was two-stepped. He first calculated what the probability was for the sex ratio being between the margins found in the London data in a representative single year. Once he had found this probability, he would simply raise it to the power of 82. This second step was straightforward, although to calculate the resulting number in all its 44 digits, as 's Gravesande did, can perhaps only be the work of one with too much time on their hands. The first step required more creativity, however. 's Gravesande first found a representative year by taking the average number of births over the 82 years in question, which was $11\,429$. For each year, he then scaled the numbers of births per sex to that average number. In this scaled data, 's Gravesande found that the number of boys had always been between $5\,745$ and $6\,128$, or between 30 and 423 more than half of the number of children born.

's Gravesande now faced the problem of calculating the probability of the number of boys out of $11\,429$ births falling between these bounds. He did by considering the expansion of the binomial $(a+b)^{11429}$. The coefficient of the term $a^n b^{11429-n}$ in this expansion expresses the number of ways in which the sequence of births can contain exactly n boys. To find the probability of the number of boys being between $5\,745$ and $6\,128$, 's Gravesande needed to sum all the coefficients of the terms $a^{5745}b^{11429-5745}$ through $a^{6128}b^{11429-6128}$, and divide this by the sum of all the coefficients in the expansion (i.e., the total number of possible birth sequences). This may seem too excessive to do by hand, and indeed better ways of estimating this distribution were soon obtained, as we shall see below. Nevertheless 's Gravesande did essentially the hand calculation, though with some shortcuts. Since the final probability is obtained as a ratio of sums of coefficients, all that matters is the relative magnitude of the coefficients. Therefore, 's Gravesande did not need to compute the coefficients in absolute numbers, but only express them in terms of the middle coefficient, that of the numbers of $5\,714$ girls and $5\,715$ boys or contrariwise, which he assigned the arbitrary value of $100\,000$. To find the other coefficients in terms of this middle term, 's Gravesande made use of a relation between adjacent coefficients, which in modern terms would be written

$$\binom{n}{x+1} = \binom{n}{x}\frac{n-x}{x+1}.$$

Using this relation, 's Gravesande calculated the individual coefficients in terms of the middle one, and collected them in a table of which the first part is shown in Figure 3. He continued the calculations down to the case of $5\,973$ boys, $5\,456$ girls, by which time the probability had dropped below 1 on 's Gravesande's arbitrary scale. 's Gravesande took the further values to be negligibly small, claiming for reasons he did not state that the sum of what we would call the tail could be no more than 50.

Now that he had the relative probabilities of all of the individual components that mattered, 's Gravesande could simply compare the sum over the relevant interval, which came to $3\,849\,150$, with the sum over the entire table, multiplied by 2 to take into account the other side of the distribution, which gave him $13\,196\,800$. Hence the probability of the number of male births falling within this range in a given year is about 0.2917, and the probability of this happening 82 years in a row



T A B L E 16.

Des cas ou des chances pour le nombre des garçons, qui naissent parmi 11429 *Enfans. Le nombre des chances pour la naissance de* 5715 *garçons, étant supposé* 100000.

Nombre des garçons.	*Nombre des chances.*	*Nombre des garçons.*	*Nombre des chances.*
5715	100000	5734	93546
5716	99965	5735	92893
5717	99895	5736	92213
5718	99790	5737	91506
5719	99651	5738	90772
5720	99454	5739	90013
5721	99245	5740	89229
5722	99002	5741	88421
5723	98725	5742	87589
5724	98415	5743	86735
5725	98071	5744	85859
5726	97693	5745	84962
5727	97285	5746	84064
5728	96843	5747	83110
5729	96370	5748	82155
5730	95865	5749	81184
5731	95330	5750	80195
5732	94765	5751	79191
5733	94170	5752	78173

Figure 3 's Gravesande's table of probabilities of different gender distributions among newborn children.

is 0.2917^{82}, or one in about $7.6 \cdot 10^{43}$: even less likely than according to Arbuthnot's calculation.

Final remarks

What can we learn from all this about 's Gravesande's mathematics? First of all, it becomes clear from these examples that 's Gravesande mastered various of the new techniques that had been developed in his day. He seems to have had a knack for finding problems that could not be solved with more conventional means, and to attack those with his heavy machinery. In both cases, however, neither the problems in themselves nor the solutions were of particular interest outside of their immediate context. 's Gravesande was not a systematic mathematician but rather a problem solver and a number cruncher. Given these examples, we are in a good position to understand one of his contemporaries, who, visiting him in Leiden in the 1720's, noted that "'s Gravesande was an industrious, skilful, but somewhat ineloquent mathematician" [12, p. 37].

's Gravesande's predilection for head-on problem solving becomes more marked when we compare him to Nicolaus Bernoulli, with whom he had a brief correspondence about the statistical problem. Initially, Bernoulli disagreed with 's Gravesande on the main conclusion. This disagreement was mostly based on Bernoulli misreading Arbuthnot's original paper, and probably of 's Gravesande's paper as well, possibly because Bernoulli had troubles with reading English and Dutch. Bernoulli believed that Arbuthnot had claimed that it was unlikely that the births would stay close to the middle term in 82 years in a row, whereas Arbuthnot had actually claimed it was unlikely that the middle term itself would pop up in any of the 82 years. In any case, this misunderstanding led Bernoulli to develop a much more general and systematic approach to the kind of statistical problem in question. Bernoulli first of all found a way to approximate the summation over the individual coefficients, thus circumventing the need for 's Gravesande's brute-force calculations. Secondly, he decoupled the notion of 'chance' from the value of $\frac{1}{2}$, and instead found that a probability of $\frac{18}{35}$ for a male birth would account for the data, including the observed variation, quite well. While Bernoulli's approach was, as 's Gravesande pointed out, besides the question with respect to the theological argument at stake, Bernoulli was able to arrive at results that were mathematically more interesting exactly because he moved away from the initialproblem.[5]

's Gravesande lived through some defining developments in the history of mathematics. He corresponded amicably with scholars of all ranks and sides, seemingly preferring Leibniz's method of infinitesimals over Newton's fluxions, notwithstanding his adoration for Newton. While being in the thick of it, 's Gravesande didn't quite fit the bill of the typical mathematician, perhaps due to the fact that he was largely self-trained. His quirkiness, however, makes him all the more interesting as an entry point in the world of early eighteenth-century mathematics. People like 's Gravesande, bringing up particular problems and dealing with them in idiosyncratic ways, created interesting opportunities for the better mathematicians to improve on their treatments. Thus, the former were catalysts to the development of mathematics itself.

Notes

1 William Burnet to Johann Bernoulli, 8 July 1710. Available online via http://www.ub.unibas.ch/bernoulli/index.php/Hauptseite.

2 G.J. 's Gravesande, Lettre sur l'Utilité des Mathématiques, in [1, Vol. 1, pp. 313–317].

3 Both of the examples I will discuss come from the beginning of 's Gravesande's career. This is a consequence of the fact that he published very little on mathematics proper during his later life. Later mathematical works of some significance are the two appendices added to [8], and annotated editions of some works by Newton and Huygens prepared by him.

4 On both Arbuthnot's and 's Gravesande's calculations, see [16] and [11, pp. 275–280]. I follow the only published version of 's Gravesande's paper, 'Démonstration mathématique du soin que Dieu prend de diriger ce qui se passe dans ce monde, tirée du nombre des garçons & des filles qui naissent journellement', in [1, Vol. 2, pp. 221–248].

5 For Bernoulli's approach see [11, pp. 280–282], for the correspondence see [2, Vol. 2, pp. 237–247].

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