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Interview Fields Medalist Martin Hairer

Intuition for mathematical concepts

On the Dutch Mathematical Congress of April 14 and 15 Fields Medalist Martin Hairer gave a plenary lecture. We took the opportunity to interview him for our magazine. Sonja Cox, researcher at the University of Amsterdam, spoke to him.

You just gave a plenary talk at the Dutch Mathematical Congress. Do you have any connections with the Netherlands?
"I attended a workshop at the Lorentz Center some years ago ('Coherent structures in evolutionary equations', July 2010), but otherwise I have never visited the Netherlands for work. I have been to Amsterdam a couple of times for holidays."

. . .and you had a Dutch post-doc.
"Yes, Jan Maas was my post-doc for a couple of months. And of course I know some people here, Frank den Hollander and Jan van Neerven for example."

You are often introduced as the son of the mathematician Ernst Hairer. But what did your mother do?
"She started off as a primary school teacher, but she stopped working when we were born. When we were grown up, she started doing a lot of volunteer work. She actually still works in a toys library, i.e., a place children can come and borrow toys."

So I assume your mother is not particularly interested in mathematics. Did your father talk to you about mathematics?
"He didn't try to push me into it, no, but he did try to answer my questions. Of course I saw his papers and wondered what it was about. I remember him trying to explain to me what a differential equation is when I was about ten or eleven. He did it graphically, showing how a formula can describe a vector field and how one can follow the lines to obtain solution paths."

You mentioned you had siblings, how many were you?
"Three, I have a younger brother and a younger sister."

Did they also study math?
"My sister also studied mathematics in Geneva. I was working on my PhD at the time that she was studying, so I heard all the gossip about how she was doing — I believe this discouraged her from continuing in mathematics, even though she was a good student. Now she has an administrative position at an eye surgery clinic in Geneva. My brother actually started off doing mathematics, too, but he switched to biology after two months. He got a degree in biology, but he really likes being outdoors, so in the end he became a sports teacher."

Did anyone inspire you mathematically when you were at school?
"Back then it was not clear that I would go into mathematics. Actually, I was much more interested in programming."

. . .but I guess you did not learn programming at school.
"No, but I got one of the first programmable graphic Casio pocket calculators for my 12th birthday. I think it could hold 422 characters, so you could only write very short programs. The most sophisticated program I made drew the Mandelbrot set, and you had to run it for a long time to get a very lousy picture. Then when I was 14 or so my parents bought a Mac, and I stuck with them ever since."

But you did not enroll for computer science when you went to university?
"Indeed, by then I had decided I knew enough about programming and I enrolled into physics. Actually, at that time in Geneva the first year of the mathematics curriculum was essentially a subset of the physics curriculum, and I continued to do more or less both programs. I did also continue to write some software alongside."

When you were at university, was there anybody whose lectures inspired you, made you feel like this was what you wanted to be doing?
"Generally I liked mathematics more than physics. Wait, that is not entirely true. I felt more comfortable doing mathematics because you can be sure if you do something and you prove it, then it is true."

But in your articles you use a lot of intuition.
"But in the end I also prove things. I *have* heard someone say that I am clearly not a true mathematician because in my articles I explain my main results in a non-rigorous fashion at first, leaving the rigorous statements and proofs for later sections. Luckily, I think most people *do* appreciate being given some intuition of what is going on before diving into the proofs."

I know I do. So there was no one in particular at university who inspired you to do mathematics?
"In Geneva at the time there was not really anybody doing analysis or stochastics in the mathematics department. Most people were working in algebra, geometry or knot

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theory, which I never quite got the hang of. In physics we had really nice classical and quantum mechanics courses, which I found a lot more interesting than most of the lectures given in the mathematics department. Another nice thing the physics department offered was something called *mathematical complements*, which was essentially a series of crash courses on various topics in mathematics. These courses provided the main results needed to apply the theory, e.g. Banach and Hilbert spaces or Schwartz distributions in two weeks, and then if you were interested you could go off and read the details and the proofs yourself. This allowed you to obtain an intuition for mathematical concepts without being burdened by having to build everything up step by step. Once you are interested and have a good idea of how things work, it is much easier to go back and understand how the theory is built up rigorously."

As a student you already realised you preferred analysis and probability theory above discrete mathematics and geometry?
"I knew I preferred analysis, I was always more comfortable with continuum objects than with discrete objects. It is a bit ironic, after all, what makes the things I have been doing recently so powerful is precisely that I use nontrivial algebra together with analysis, and this is really interesting. However, I did not know any probability theory as it was not taught in Geneva at the time. As a student, I thought probability theory was limited to urn and card shuffling games and I had no idea of the analytical side of probability theory. This I only discovered during my PhD."

Do you feel your taste for mathematics shifts?
"I think so, yes. You need motivation to get interested. Most areas are interesting, but you need some motivation to look at them. But taste does shift."

Can you describe when a problem motivates you?
"I think it is a problem of which you feel on the one hand that you really *should* be able to understand it, but on the other hand... you don't. In other words, there is a dissonance between what you think you should understand and what you actually understand, and you think 'It can't be that hard...' but it is."

Of all your work so far, what is your favorite?
[laughs] "Well, okay, so I think it's the regularity structures paper. First, it touches the most different fields of mathematics; there is lots of analysis, some probability theory, and some algebra. But it also creates a new field. Probably my second favorite is the work with Jonathan Mattingly on the ergodicity of the stochastic Navier–Stokes equation."

When did you start working on the regularity structures?
"It was gradual; I'd been thinking about these types of problems for some time. It really started with my work with Andrew Stuart on the problem of finding a stochastic partial differential equation (SPDE) the invariant measure of which is the solution to a given stochastic (ordinary) differential equation (SDE). If the SDE simply has additive noise and a gradient drift, then you get a reaction-diffusion type SPDE which is well understood. However, if the drift is not a gradient, you end up with an SPDE that is ill-posed. This is strange, as the SDE is perfectly well-posed. To me this was exactly one of those problems of which you feel you *should* be able to understand them, but you don't."

What do you think are interesting developments in your field at the moment?
"In probability theory there are quite a few very interesting recent developments. I think one of the most interesting is what Scott Sheffield has been doing with Jason Miller. They have a series of papers in which they build a two-parameter family of processes that generalise Schramm–Loewner evolutions (SLEs) and that they call 'quantum Loewner evolutions' (QLEs). These processes are not random curves, but random increasing families of subsets of the complex plane with intricate fractal structures and beautiful mathematical properties, like conformal invariance. For one particular value of the parameters, this process is conjectured to describe the image, under a conformally uniformizing map, of the ball of radius t centred at some fixed point in the 'Brownian map' constructed recently by Miermont and Le Gall. For another value of the parameters, it is conjectured to describe the scaling limit of Diffusion-limited aggregation."

Suppose you were allowed to do neither research nor teaching for a whole year, what would you do?
"I would probably go on some big trip around the world and see some jungles and mountain tracks and stuff like that, and take a few good books with me... I'm still allowed to read non-math books, right?"

Yes of course. You are also allowed to study chemistry, if you like. Just no mathematics.
"Oh, if I'm still allowed to actually study something, then... [long pause] yes, probably I'd rather take some sociology books or something like that. Something more toward human science, just to do something else."

And you would prefer sociology above, say, psychology?
"Yes. Yes, I am just wondering whether there is something interesting to say there. It may well be that after two months of reading I would decide there isn't anything interesting for me. That's possible. But maybe not."

Your wife Xue-Mei Li works as a mathematician in Warwick. Did you meet there?
"Yes, but at the time neither of us was working there. She is basically the reason I moved to Warwick. We met at a workshop in Warwick which was part of a semester program on SPDEs organised by David Elworthy. Xue-Mei is a former PhD student of David, and at that time she had a position in Nottingham."

...and you got together at that workshop?
"Yes. I think that's a well-known secret by now."

Do you talk a lot about math at home?
"Occasionally, but we try to talk about other things as well."

Second-last question: what would be your advice to a student who wants to do a PhD in mathematics?
"Choose your PhD supervisor carefully. That is probably the most important thing. It is not necessarily important that the supervisor is a very famous person, but he should be doing something that you are actually interested in. When you choose your topic, you should think about what you are interested in and what you are comfortable with, instead of jumping onto the latest hype."

Last question: is there anything you would ask yourself that has not been asked yet?
"Oh, I don't know about that one. I think you already have quite a big selection of questions. No, I am not sure I can think of anything."

Well, thank you very much for the interview.