David Mumford (1937) was active in algebraic geometry in the period 1961–1984. His contributions have an enormous impact on this field. Then he began a new scientific life in the field ‘biology and psychology of vision’. This is a rare occasion where a mathematical production in one field is considered final whereas the author still is very active (in another field). With this volume (after Volume I) we have a complete reproduction of all papers in algebraic geometry by David Mumford.

Volume I of Selected Papers by David Mumford appeared in 2004 (Springer). A nice review by János Kollár appeared in the Notices AMS 43 (2005), pp. 111–114. In that volume part of his papers was reprinted. However, the selection seemed not very consistent. For example, that volume was ‘on the classification of varieties and moduli spaces’, but the paper with Deligne, ‘The irreducibility of the space of curves of a given genus’ (1969), with 325 citations one of the most influential papers in modern algebraic geometry, was lacking.

In Nieuw Archief voor Wiskunde, 5/8 (2007), pp. 225–227, we noted some inconsistencies in the first volume, such as: “It was not clear why some papers were not reproduced and the papers did not appear in chronological order. That volume contains five different lists of references. No two agree. There are painful mistakes such as page numbers that are omitted or wrong; names that are misspelled; references that are unsystematically abbreviated, even within the same list...” The present volume gives all papers not appearing in the first volume: now we have the algebraic geometry papers by Mumford complete.

This present volume corrects flaws of the first volume. In Volume II we see a complete and precise list of books and papers written by Mumford; this list truly can be trusted. Moreover, the editors have annotated these papers with footnotes, commenting on data not available when the paper considered was written, correcting some misprints, indicating new developments. This is a great service to the mathematical community.

Next to giving these papers by Mumford, this volume publishes (part of a) mathematical correspondence between Grothendieck and Mumford, plus several letters from Grothendieck to other mathematicians. The editors have done a great job in editing and typesetting these letters, in adding more than 160 footnotes explaining ideas in those letters, and providing recent developments and references. This is a valuable source of information. It gives us insight in the exchange of ideas between these two towering figures in modern algebraic geometry.

For example, Mumford wrote [61c], ‘An elementary theorem in geometric invariant theory’, a paper which takes up classical invariant theory and provides new insight and constructions; the numbering [61c] refers to the list of publications of Mumford; this is the third paper in 1961. On receiving a preliminary version of this paper Grothendieck was impressed, as we see in his letters in the period April 1961 – January 1962 (pp. 634–641); interesting reading, where we can compare the methods by Mumford, and the immediate reaction by Grothendieck. Another example: the paper by Mumford on ‘Abelian quotients of the Techmüller group’ [67d] was received by Grothendieck...
and he replied by writing: “As a general impression, I found it astonishing that you should be obliged to dive so deep and so far in order to prove a theorem whose statement looks so simple-minded” (9 May, 1966, page 717). This unique document enables us to compare a paper of Mumford and the immediate reaction of Grothendieck on receiving an early version of that paper. What a wonderful access to the difference in style of thinking and working in algebraic geometry. Also we see the great respect they both had for each other.

Grothendieck tries to prevent publication of material from his hand which did not appear before. However, the editors in time were able to secure permission for publication of these letters (see pages v and xii). Also the editors had not an easy road because of some difficulties with the publishing company, see www.math.upenn.edu/~chai/story/story.html. However, the present volume shows no traces of these obstacles, overcome in a wonderful way. It is a pleasure to read these papers again, to see manuscripts that were not easily accessible, and to read discussions in this correspondence. A valuable and beautiful monument to one of the main figures in algebraic geometry of the past century.

Frans Oort

My first reaction to this book was: “Wait a moment — 600 pages about the calculus of variations, and no derivatives?” Indeed, that’s exactly what they do.

The book is divided up as follows. The first 130 pages are devoted to measure theory, and include both basic tools such as the covering theorems and differentiation and less-often seen results such as the projection of measurable sets. There is also an extensive discussion of the weak-star topology on the space of measures.

Chapter 2, on \( L^P \)-spaces (80 pages) covers everything one would expect and then some, including for instance the Biting Lemma, maximal functions, and vector-valued \( L^P \)-spaces. Chapter 3 is a brief discussion of the ‘direct method’ of the calculus of variations. In Chapter 4 (80 pages) the authors introduce properties of convex sets and convex functions, including e.g. the concept of relative interior, the regularity of convex functions, polar and bipolar functions, and approximation properties. As the authors state in the Preface: “The core of this book is the analysis of necessary and sufficient conditions for sequential lower semi-continuity of functionals on \( L^P \)-spaces, followed by relaxation techniques.” This core takes place in Chapter 5 (60 pages) on functionals of the form \( \int f(u(x))\,dx \), in Chapter 6 (80 pages) for integrands of the form \( f(x, u(x)) \), and in Chapter 7 for integrands of the form \( f(x, u(x), v(x)) \). Chapter 8 covers the topic of Young measures.

The style is that of a reference work, not of something one might read sequentially. Proofs are complete, going into all detail. Unfortunately, this leads to much referring: a theorem in Chapter 7 might use a result from Chapter 4, which itself refers to a remark in Chapter 2, which elaborates on a theorem in the same chapter. It is good if you want all the nitty-gritty, and are willing to flip back and forth through the book. Luckily, the notation struck me as completely standard, and I had little difficulty jumping into a new section and picking up the discussion from there on.

One aspect that I feel is lacking is the explaining why a theorem really is true. One example out of many is the equivalence of convexity of a real-valued function \( f \) and the weak lower semi-continuity of the functional \( f(u) \) in \( L^P \), which is one of the central results in this book. In my mind the real reason why convex functions make for weak lower semi-continuous functionals is the property that the function is larger than the tangent,

\[
\forall y \exists p(y) : \forall z \ f(z) \geq f(y) + p(y) \cdot (z - y),
\]

since this implies

\[
\int f(u_n(x))\,dx \geq \int f(u(x))\,dx + \int p(u(x)) \cdot (u_n(x) - u(x))\,dx.
\]

Since the final integral vanishes when \( u_n \to u \), the inequality follows. This is not a proof, since the function \( p(u(\cdot)) \) need not lie in the appropriate \( L^1 \)-space (indeed it need not even be measurable) and consequently the proof given by Fonseca and Leoni contains a significant amount of technical supporting arguments. But it would have been nice to hear that the property just described is the core of the proof.

The reference-like style of this book is probably intentional. For instance, in the Preface the authors state: “We believe that this text is unique as a reference book for researchers, since it treats both necessary and sufficient conditions for well-posedness and lower semi-continuity of functionals, while usually only sufficient conditions are addressed.” In my opinion, the authors have a valid point here, and the discussion of full equivalence struck me as refreshing.

This book has certainly earned a place as a reference work in my bookcase. I won’t recommend it to any of my students for light reading; but I am sure I’ll pick it up every once in a while to look up the details of some argument. And I’m intrigued to see what the second volume will bring.

Mark Peletier

Irene Fonseca, Giovanni Leoni
Modern Methods in the Calculus of Variations: \( L^P \)-spaces
Springer, 2009
Springer Monographs in Mathematics
XIV + 600 p., prijs € 58,25
ISBN 9780387357843

My first reaction to this book was: “Wait a moment — 600 pages about the calculus of variations, and no derivatives?” Indeed, that’s exactly what they do.

The book is divided up as follows. The first 130 pages are devoted to measure theory, and include both basic tools such as the covering theorems and differentiation and less-often seen results such as the projection of measurable sets. There is also an extensive discussion of the weak-star topology on the space of measures.

Chapter 2, on \( L^P \)-spaces (80 pages) covers everything one would expect and then some, including for instance the Biting Lemma, maximal functions, and vector-valued \( L^P \)-spaces. Chapter 3 is a brief discussion of the ‘direct method’ of the calculus of variations. In Chapter 4 (80 pages) the authors introduce properties of convex sets and convex functions, including e.g. the concept of relative interior, the regularity of convex functions, polar and bipolar functions, and approximation properties. As the authors state in the Preface: “The core of this book is the analysis of necessary and sufficient conditions for sequential lower semi-continuity of functionals on \( L^P \)-spaces, followed by relaxation techniques.” This core takes place in Chapter 5 (60 pages) on functionals of the form \( \int f(u(x))\,dx \), in Chapter 6 (80 pages) for integrands of the form \( f(x, u(x)) \), and in Chapter 7 for integrands of the form \( f(x, u(x), v(x)) \). Chapter 8 covers the topic of Young measures.

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Bernard Vitrac
Meetkunde in het klassieke Griekenland
VBK Media, 2012
191 p., prijs € 42,50
ISBN 9789085731347

Dit is een erg mooi boek van de hand van een Frans historicus over wiskunde en wiskundigen in de Klassieke Oudheid, waarvan helaas de vertaling uit het Frans veel te wensen overlaat.

Laat ik beginnen met de positieve kanten. Het boek is een aangezien, fraai geïllustreerd mix van wiskunde en de vele aspecten van de context waarin de zuivere wiskunde is ontstaan en zich heeft ontwikkeld. Ik ken geen ander werk waarin politieke, culturele, biografische, anekdotische en vaktuinse facetten van de Griekse wiskunde zo mooi samen worden behandeld. Het boek begint met de raadselachtige oorsprong van de Griekse meetkunde, vervolgd met het belangrijke pre-euclidise werk van Hippocrates om daarna aan de hand van De Elementen van Euclides verschillende aspecten te behandelen van de meetkundige traditie die we
The subject of dynamical systems came into existence with the input of Poincaré around 1900. Till then the tool-kit for studying motion was held; four is enough. Eudoxus is in the book the bedenker of the ‘uitputtende methode’, ‘meetkundige plaats’ becomes in the book ‘plaats’ genoemd, de maantjes van Hippocrates worden de ‘boltwee-hoeken’ van Hippocrates genoemd en er is tenminste één keer sprake van een ‘onwaar bewijs’. The auteur of the book, Bernard Vitrac, has as Fransman een enigszins retorische stijl. De vertaler is er ook niet in geslaagd om dit goed te hanteren en de vertaling leest niet zelden moeizaam. Dat is allemaal erg jammer, want Vitrac heeft echt een mooie tekst geschreven. De oorspronkelijke tekst is in het Frans op het web te vinden (www.math.ens.fr/culturemath/histoire). U kunt misschien ook wachten op een Engelse vertaling.

Johannes J. Duistermaat
Discrete Integrable Systems
QRT Maps and Elliptic Surfaces
Springer Verlag, 2010
Springer Monographs in Mathematics
XXII + 627 p., prijs €116,55
ISBN 9781441971166

The dynamical interest is with iterates of such rational transformations in the affine real plane, the formulae of which get very messy. It turns out that the results get far better organized and more uniform when complexifying and compactifying to the product of two complex projective lines. As the author puts it: “The point is that in the complex projective setting the full force of complex projective algebraic geometry is at our disposal.” Chapter 6 consists of 150 pages on elliptic surfaces, followed by 50 pages of Chapter 7 devoted to automorphisms of such surfaces.

In the preface the author names three possible groups of readers: those from discrete dynamics, the people from discrete integrable systems and the readers with a background in algebraic geometry. The book is devised to serve all three groups in some way or another and its more than 600 pages aim at a fair amount of self-containedness. The book moreover contains a clearly written introduction, a bibliography of over 200 references and an extensive index. I found the book a monument of culture and a clear expression of the author’s great erudition. Therefore it is highly recommended.

Henk Broer

Barry Koren, Kees Vuik (eds.)
Advanced Computational Methods in Science and Engineering
Springer, 2010
Lecture Notes in Computational Science and Engineering 71
X + 498 p., prijs €132,45
ISBN 9783642033438

Computational methods are of paramount importance for the simulation of many processes and systems in science and engineering. Nowadays, many disciplines have their own computational branch, next to a theoretical and an experimental branch. Examples are Computational Fluid Dynamics and Computational Electromagnetism, and there are many more. All these disciplines are collectively referred to as Computational Science and Engineering (CSE). This text gives an interesting overview of CSE research carried out by scientists of the Delft Centre for Computational Science and Engineering. To mention just a few examples, topics that are covered are model-order reduction for electromagnetic problems, numerical methods for chemical vapour deposition, finite volume methods for hyperbolic equations and large eddy simulation for combustion problems, but there are many more. Each chapter describes one topic and contains cutting edge research, which is often also available as journal publication. Key in most chapters is the description of numerical methods underlying computational techniques, but also numerical experiments for complex problems are presented. The book gives a nice impression of CSE research at the Delft Centre for Computational Science and Engineering and is interesting for everybody involved in CSE somehow.

Jan ten Thije Boonkkamp
Recent verschenen publicaties. Als u een van deze boeken wil bespreken of als u suggesties heeft voor andere boeken voor deze rubriek, laat dit dan per e-mail weten aan reviews@nieuwarchief.nl.

Eric Lord
Symmetry and Pattern in Projective Geometry
Springer, 2013
ISBN 9781447146308
www.springer.com/978-1-4471-4630-8

Glen Van Brummelen
Heavenly Mathematics
The Forgotten Art of Spherical Trigonometry
ISBN 9780691148922
press.princeton.edu/titles/9834.html

Julian Havil
The Irrationals
A Story of the Numbers You Can’t Count On
ISBN 9780691143422
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Odo Diekmann, Hans Heesterbeek, Tom Britton
Mathematical Tools for Understanding Infectious Disease Dynamics
ISBN 9780691155395
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Martin Kindt, Ed de Moor
Wiskunde, dat kun je begrijpen!
Uitgeverij Prometheus/Bert Bakker, 2012
ISBN 978903513856
uitgeverijprometheus.nl/?option=com_pac&view=boek_detail&isbn=978903513856

Wolfram Decker, Gerhard Pfister
A First Course in Computational Algebraic Geometry
Cambridge University Press, 2013
ISBN 9781107612532
www.cambridge.org/978-1-107-61253-2

Shigeru Mukai
An Introduction to Invariants and Moduli
translation: W.M. Oxbury
Cambridge University Press, 2012
ISBN 9781107029804
www.cambridge.org/978-1-107-02980-4

Francis Buekenhout, Arjeh Cohen
Diagram Geometry
Springer, 2013
ISBN 9783642344527
www.springer.com/978-3-642-34452-7