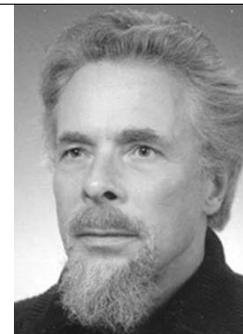


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Floris Takens

## In Memoriam Floris Takens (1940–2010)

# A total mathematician

On June 20, 2010, the Dutch mathematician Floris Takens passed away at age 69. This article presents the text of the eulogy by Henk Broer at the funeral of Floris Takens. Besides that it contains remembrances by David Ruelle and Freddy Dumortier.

Els, Leida, Vincent, other family members, friends, colleagues and acquaintances!

### The scientist

Floris Takens became a full professor of mathematics at Groningen University in 1972, at the age of 31 years. He had gained his doctorate at the University of Amsterdam in 1969 un-

der the supervision of Nico Kuiper. His thesis was entitled The minimal number of critical points of a function on a compact manifold and the Lusternik-Schnirelman category. Following his doctoral studies he spent a year as a guest researcher at the Institut des Hautes Études Scientifiques in Bures-sur-Yvette near Paris (1969–1970). Here he was influenced

by both René Thom and David Ruelle (known in Groningen from the 1999 Johann Bernoulli Lecture). With Ruelle he wrote the paper ‘On the Nature of Turbulence’, published in the journal *Communications in Mathematical Physics* (1971) [1]. This was a groundbreaking paper, in which a new idea was introduced contradicting the established theory about the onset of turbulence in fluid motion as developed by the leading physicists Landau and Lifschitz and the eminent mathematician Hopf. The new idea proposed by

### Biography of Floris Takens

Floris Takens was born 12 November 1940 in Zaandam, the third of four children. His parents, Willy Bremmer and Pieter Roelf Takens were both teachers of Latin and Greek. Floris had two brothers, Henk and Roelf, and a sister, Leida. Floris was named after his uncle Floris Bremmer, who lived in The Hague. Bremmer in turn had been named after the painter Floris Verster, who was a colleague and good friend of Floris’s grandfather H.P. Bremmer. Floris was in possession of one of his paintings.

As a boy, Floris lived with his uncle and aunt for about 5 years. He started his secondary school in The Hague at the Maerlant Lyceum (HBS). While at school Floris started playing the flute, which became a lifelong passion. Apart from artists, there were also scientists in the Bremmer family, such as Floris’s uncle Henk Bremmer, who worked at Philips NatLab and at the Eindhoven University of Technology. The home environment at the

Bremmers inspired Floris a lot and encouraged the development of a very strong work ethic. It was during these years that not only science, but also painting and music became important to him.

Upon returning to his parental home in 1957, he continued his secondary schooling at the Zaanlands Lyceum. During this time he joined the Zaans Jeugdorkest ‘Jeugd en Muziek’ (a youth orchestra). This period, and also later at the university, saw the development of an enduring interest in baroque music which began with his enrollment at music classes at the ‘Volkshogeschool Bergen’, quite near Zaandam. At school he was both ambitious and self-disciplined, a necessary trait in view of his dyslexia. Young Floris also showed a lot of talent and ingenuity by building all kinds of objects in his free time, such as radios and model airplanes.

Floris went to the University of Amsterdam in 1959. His studies were interrupted dur-

ing 18 months for military service, where he took a great deal of trouble to find a studio (in an attic) for his flute practice. Once back at the university, he became a member of the ‘Zaanse Studentenvereniging’, where he really learned to party. He took several administrative duties in the society, and became a member of the debating society ‘Aromara’ where he established a number of life long friendships. His stay at the University of Amsterdam was concluded with his PhD in 1969. Floris had married Janna Vera Dijk in 1965, and in 1967 their daughter Els was born. The marriage ended in 1993. From 1972 till 1999 Floris was professor at the University of Groningen. He was extremely successful as a scientist and he established a school in the theory of Dynamical Systems. For more details see the main text. Floris Takens died on 20th of June 2010 at the age of 69 in Groningen. He is buried at the cemetery in the nearby village of Bedum, his home for the last 20 years of his life.

Ruelle and Takens was baptized ‘strange attractor’, this was later incorporated into the theory of chaos.

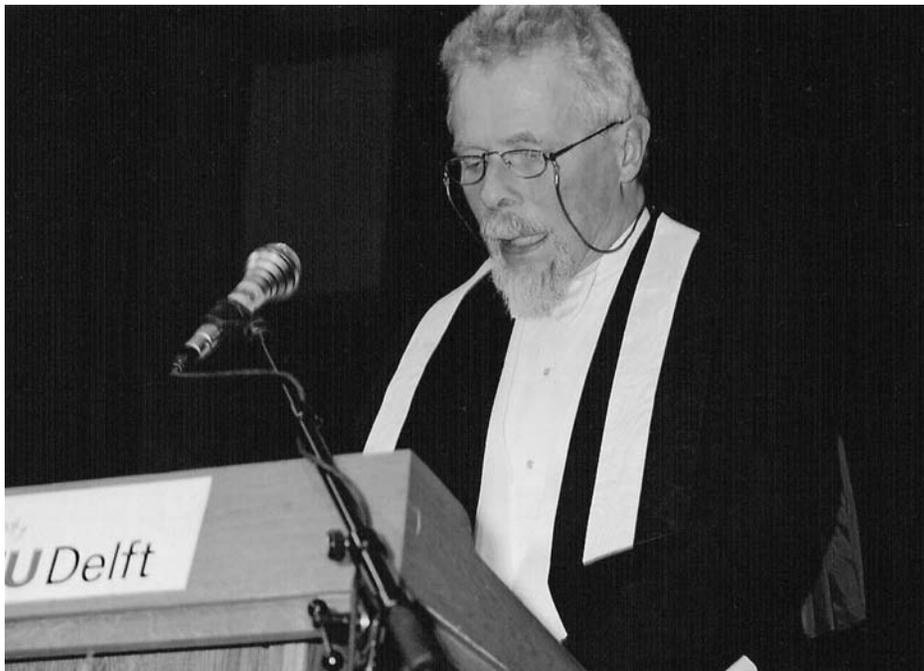
### Context

Takens’s chair was in the field of ‘Differential Topology, in particular Dynamical Systems’. Geometrical ideas had been introduced into dynamical systems research by the pioneering work of Poincaré, among other things in celestial mechanics. In the 1960’s and 1970’s an enormous impetus was given in this direction by the input of the Fields Medal winners Stephen Smale (University of California Berkely) and René Thom (IHES) and their schools. Both had become famous in geometry (topology). Thom, moreover, became renowned for the introduction of ‘Catastrophe Theory’, which later became somewhat controversial. The former topologist Christopher Zeeman (Warwick) also participated in the application of this theory (Zeeman held the 1993 Johann Bernoulli Lecture). This was a culture which suited Floris Takens perfectly. The Brazilian Jacob Palis, who obtained his Berkeley doctorate from Smale, is of the same generation. Floris and Jacob maintained an extensive and extremely fruitful collaboration from 1971 onwards. For a long period Floris was a guest researcher for several months each year at the beautifully situated Instituto de Matemática Pura e Aplicada in Rio de Janeiro.

### Research themes

Floris Takens wrote dozens of important papers, many of which are influential to this day for researchers all over the world. Roughly speaking two directions can be distinguished in his work, taken together he supervised about 20 PhD students in these areas.

*Stability, hyperbolicity, bifurcations.* With his contributions to structural stability and moduli in the setting of (almost) hyperbolicity and the bifurcations from simple to complex behaviour, Takens is surely one of the founding fathers of the modern discipline of Dynamical Systems. His major scientific stature and his numerous international contacts have greatly benefited both the University of Groningen and the Dutch mathematics community as a whole. His PhD students in this scientific direction at Groningen University were Albert Hummel, myself, Gert Vegter, Fopke Klok, Jan Barkmeijer, Cars Hommes, Ale Jan Homburg, Bernd Krauskopf, Florian Wagner, Evgeny Verbitskiy and Renato Vitolo. Outside Groningen we can add Freddy Dumortier, Bert Jongen and Sebastian van Strien



In 2001 Floris Takens became a honorary doctor at Delft University

to this list.

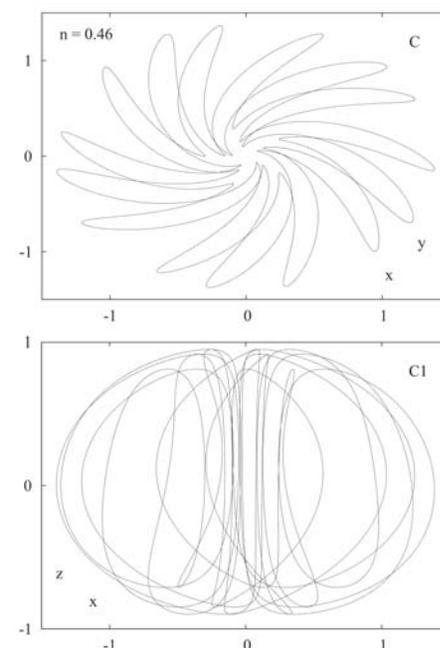
*Nonlinear time series.* Around 1980 Floris Takens initiated a new direction in which information can be obtained regarding characteristics of the dynamics, such as dimensions of attractors, entropy, Lyapounov exponents, et cetera, from time series generated by deterministic systems where the equations of motion need not be known [2]. Many nonmathematicians have applied and adapted this theory, currently known as the ‘Takens Reconstruction Theory’. His contributions to chemical process technology earned him an honorary doctorate at Delft University of Technology, an award in which he took considerable and justified pride. In this research direction he acted as an advisor for a number of external PhD students, namely Jan-Pieter Pijn, Pieter Been, Cees Diks and Marcel van der Heijden. In Groningen he was also co-advisor of Svetlana Borovkova.

### Total mathematician

For Floris Takens the discipline of mathematics was one organic entity, including the applications. This fits well with his own career, in which both the ‘pure’ differential topology and the ‘applied’ time series analysis coexisted in a brotherly fashion. In his papers, among other things, analysis, geometry (in many manifestations) and measure theory take their natural place. Furthermore he wrote programs himself in computer languages such as Matlab and C++ when the need arose. It should be noted that the area

of dynamical systems is closely connected to mathematical physics, as is apparent from Takens’s early work with Ruelle.

Floris always fiercely resisted the constant threat of fragmentation of the mathematics curriculum. One of his ideals was that all professors would be able to teach all courses in the first three years of the curriculum (corresponding to the Bachelor curriculum). This never came to pass in Groningen, but I’m sure that he himself would have been able to do this without any problem.



Multiperiodic and strange attractor

When we remember a fellow scientist, it is natural to think of the honors received at the end of his or her career, and one gets thus the image of an important elderly person. This is however not how I like to think of Floris Takens. I interacted with him when we were both fairly young, and preoccupied with scientific problems — understanding the nature of things — rather than with the thought of becoming important academicians. I would thus like to evocate the carefree atmosphere of the discussions that led us to write ‘On the Nature of Turbulence’. Floris and I had a common interest in hyperbolic dynamical systems, about which René Thom and Steve Smale had been lecturing at the IHES. I also had spent some time trying to learn about hydrodynamics from the book of Landau and Lifshitz. I was however dissatisfied with Landau’s interpretation of turbulence as a quasi-periodic time evolution resulting from successive Hopf bifurcations affecting different ‘modes’ of a fluid. A quasi-periodic time evolution is in fact not structurally stable: Floris and I thought that quasi-periodic dynamics could be perturbed into dynamics with a ‘strange attractor’, one of the strange objects discussed by Steve Smale, that looked a lot more turbulent than quasi-periodic motion. We succeeded to show how strange attractors could indeed appear by perturbation of quasi-periodic motions (this would later be called the quasi-periodic pathway to chaos); we wrote a paper on this and submitted it for publication. The paper was rejected, but since I also was a journal editor, I refereed it and accepted it

without further ado. This paper ‘On the Nature of Turbulence’ is apparently the first place where the phrase ‘strange attractor’ appears in print. People later asked Floris and me which one of us had put the two words together. The answer is that neither of us remembered. This is what I have in mind when I speak of our carefree attitude, as is the fact that I refereed and accepted for publication a paper of which I was a co-author (very, very bad!). Our paper ‘On the Nature Of Turbulence’ attracted more attention than Floris and I expected but, to our surprise, many people thought that it was wrong! This was in fact excellent for us because we didn’t have to wage priority fights to defend the paternity of our ideas. (The important work of Ed Lorenz was acknowledged when it became known outside of the meteorological community.) Little by little the idea of strange attractors in physics became popular, and it received the name of ‘chaos’. Floris continued to work on the subject by studying the reconstruction of dynamics from time series. I worked in other directions, and we no longer collaborated scientifically. But I would see Floris regularly at the IMPA on Rio de Janeiro. Here is the image that I shall keep of him: Floris is standing with a glass of batida in his hand, and a grin on his face, and speaks at leisure with his hearty Dutch accent. The question he discusses may be mathematical or non-mathematical, anything in fact that has caught his fancy. Whatever it is, you listen, because what he says is intelligent, deeply thought, and up to the point.

David Ruelle

Takens was an editor of the *Springer Lecture Notes in Mathematics*, an honorable task which he continued to perform for a further decade after his retirement in 1999. Bernard Theissier, one of his fellow editors, sent a condolence message highly commending Floris’s ‘immense culture’.

After his retirement Floris Takens was still closely involved with the PhD theses of Renato Vitolo (2003, here he acted as a co-advisor) and of Olga Lukina (2008). One of his interests in the last decade of his life consisted of the geometry of torus bundles as these occur in integrable and nearly integrable Hamiltonian systems. This is an interesting area of research related to earlier work by Hans Duistermaat and Richard Cushman, having both

classical and quantum-mechanical applications, among other things in theoretical chemistry. Here he could whole-heartedly give free rein to his old passions for differential geometry and algebraic topology. In addition Floris faithfully attended PhD defenses, colloquia and relevant seminars. Together we worked on an advanced text book *Dynamical Systems and Chaos* [3] and on the *Handbook of Dynamical Systems Vol. 3*. [4]

#### Additional activities

Apart from numerous teaching duties among which the famous (and somewhat dreaded) bachelor course ‘Differentialrekening in  $\mathbb{R}^n$ ’, Floris Takens regularly taught master courses in analysis on manifolds, differential geom-

etry, differential and algebraic topology, and he held weekly seminars in his office. These dealt mostly with dynamical systems, but occasionally also with Riemann surfaces, sheaf theory, and many other subjects.

Floris also performed his share of administrative tasks. From around 1990 he served for several years as the chairman of the math department. Also he served a term as chairman of the national Mathematics Research Institute (jointly with the Universities of Nijmegen, Twente and Utrecht). Floris was one of the founders of the Dutch FOM/SWON program Mathematical Physics and also acted as chairman for a period. In 1991 he became a member of the Royal Netherlands Academy of Arts and Sciences (KNAW), while much earlier he was already made a member of the Brazilian Academy. Within the KNAW he was also involved in administration, among which the chair of the mathematics section.

Floris Takens never shirked the demanding tasks that came his way. An example is the inter-university teaching assessment that took place in 2008 in both the Netherlands and Flanders. When the acting chairman Jacques van Lint suddenly passed away in medias res, he took over this responsibility.

#### The ‘mensch’

Floris had a reputation for being a meticulous man with a strong sense of duty and consistently high standards. This applied both to his daily practice of the flute as well as his precision in all matters at the department or elsewhere. He used to arrive at work with a Spartan punctuality every morning, whether or not he had partied long and hard the previous night. I remember numerous joyful open-air sessions in Rio de Janeiro and in Trieste, with a view on the Atlantico or the Adriatico and a table filled with empty bottles: eat, drink and be merry, meanwhile talking about life itself.

As we have seen, Floris never shirked his duties but rather fought hard for his beliefs, not always effectively. In fact, his attitude was a bit soldierly, stoically accepting one’s responsibilities and doing one’s duty without complaint. In return he could not bear tardiness in others. It must be said that Floris was not always an easygoing person, neither for himself nor for others. Over the past twenty years I remember Floris to an increasing degree as a good colleague and a warm friendship grew between us. This also had to do with the change in his personal circumstances, finding himself in calmer waters after his move to the village of Bedum.

**Floris Takens, promotor en vriend**

Toen ik in 1969 aan de Vrije Universiteit Brussel afstudeerde als licentiaat wiskunde, leek het mij dat een Banachruimte zo ongeveer de eenvoudigste structuur was waar nog iets interessants over te onderzoeken viel. Het was dan ook de bloeiperiode van de oneindig-dimensionale differentiaaltopologie. In 1970 trok ik, met enkele resultaten en ideeën, naar Nicolaas Kuiper in Amsterdam, met de hoop hem te kunnen overtuigen om mijn promotor te worden. Hij stelde toen voor dat ik zou gaan werken onder de leiding van één van zijn oud-studenten, net terug van een verblijf in het IHES en klaar om een promovendus te begeleiden: Floris Takens. Floris was een exponent van zijn tijd, met lang haar en altijd op sandalen. Bij meer protocollaire aangelegenheden kon men hem wel eens zien in toga, weliswaar met een jeans eronder (en sandalen). Floris heeft trouwens nooit veel aandacht besteed aan kledij. Kledij was iets functioneels: eenvoud en netheid waren het belangrijkste.

Floris stelde me voor te gaan werken aan vlakke vektorvelden, meer bepaald aan singulariteiten van dergelijke systemen. OK, het betrof een twee-dimensionale Banachruimte, maar in het begin ontging me toch nog de interesse en moeilijkheidsgraad van het probleem. Gelukkig bezorgde Floris mij heel wat recente literatuur betreffende dynamische systemen en kwalitatieve studie van differentiaalvergelijkingen. Allemaal nieuw voor mij.

Hij bezorgde me ook een methode van aanpak: desingularisatie door middel van herhaaldelijke blow-up (gebruik van poolcoördinaten). Zelf was hij toen door middel van blow-up de singulariteiten van meerdimensionale vektorvelden aan het bestuderen tot codimensie twee. Deze resultaten heeft hij in 1974 gepubliceerd in het veel geciteerde artikel 'Singularities of vector fields' (*Publ. Math. IHES*). Dit artikel was heel zeker een inspiratiebron voor heel wat onderzoekers in het domein: blow-up methode, gedetailleerde aanpak van invarian-

te variëteiten en normaalvormen, een eerste voorbeeld van een niet-stabiliseerbare jet.

De problematiek was geïnspireerd door de catastrofentheorie van R. Thom en was gericht op de uitbreiding van deze theorie tot de studie van singulariteiten van vektorvelden in plaats van kritieke punten van functies. Daarbij leek het logisch om eerst een beschrijving te geven van de singulariteiten in functie van stijgende codimensie, om daarna de ontvouwing van deze singulariteiten te bekijken. Zo kon men komen tot meer 'dynamische catastrofes', met andere woorden stabiele bifurcaties afhankelijk van een klein aantal parameters. De gebruikte equivalenties, om stabiliteit te omschrijven, waren wel topologisch en dus minder rigide dan bij functies, maar toch doken al heel snel subtiliteiten op die Floris stelselmatig onder de aandacht bracht. Floris hield er van om de bewijzen tot op het bot uit te pluizen en van de nodige technische details te voorzien. Indien bestaande normaalvorm stellingen niet helemaal voldeden dan maakte hij er betere.

Voor mij was hij de ideale promotor: niet in mijn nabijheid als ik behoefte had om rustig op mijn eigen tempo te werken, maar klaar om op mijn vragen te reageren en om mijn resultaten gedetailleerd te controleren. Hij beantwoordde stipt mijn brieven, tussen twee reizen in bleef hij soms enkele uren in een station in Brussel om met mij te praten, en wanneer de tijd rijp was kon ik bij hem thuis logeren om intensief mijn resultaten met hem door te nemen.

Hij heeft mij in een vakgebied gelooft dat me tot vandaag blijft boeien en waar ik me nog steeds laat inspireren door zijn resultaten en methoden van aanpak. De zoektocht naar ander voorbeelden van niet-stabiliseerbare jets heeft me beziggehouden tot ik ook een tegenvoorbeeld in drie dimensies vond. Zijn artikelen 'Forced oscillations and bifurcations. Applications of global analysis' (1974) en 'Unfoldings of certain singularities of vector fields: generalized Hopf bifurcations' (1973) hebben

mij, en met mij vele anderen, geïnspireerd om deze studie verder te zetten naar 2-dimensionale ontvouwingen van hogere codimensie of ook naar bifurcaties in een hogere dimensie.

De aanpak sloeg duidelijk aan en kon zelfs mensen beroeren die tot dan toe niet direct appreciatie hadden voor die 'generieke lokale bifurcaties'. Met de nodige normaalvorm berekeningen kon aangetoond worden dat deze lokale bifurcaties de facto optreden bij relatief eenvoudige wiskundige modellen in een variëteit van wetenschappelijke domeinen. Heel wat onderzoekers lieten zich dan ook verleiden om hier aandacht aan te besteden, hetzij in de theoretische uitwerking van meer complexe bifurcaties, hetzij in de lokalisatie van deze bifurcaties in ingewikkelder modellen. Een referentie naar Takens is dan ook terug te vinden in heel wat proefschriften, zowel van zuivere als van toegepaste wiskundigen, en niet alleen van promovendi die in directe lijn tot Floris terug te voeren zijn. De toepasbaarheid van de theorie werd nog versterkt door het gebruik van accurate numerieke technieken, waarbij Floris, door tussenkomst van studenten, ook een rol in gespeeld heeft. Ik wil hier nu niet de indruk wekken dat Floris Takens de enige voedingsbodem was voor vernieuwende ideeën in het domein. Dergelijke uitspraak had Floris zelf me zeker kwalijk genomen. Maar hij was ongetwijfeld een van de belangrijkste inspiratiebronnen.

Op Floris kon je altijd rekenen. Een vraag tot geven van een voordracht of een lessenreeks werd stevast positief beantwoord. Hij was strikt en correct, maar was ook genereus en enthousiasmerend. We hebben vele gezellige avonden met elkaar doorgebracht, in allerlei exotische oorden, maar ook bij hem thuis of bij mij thuis. Hij was een levensgenieter, maar een zeer belezen en erudiete levensgenieter. Ik mag mij in alle opzichten gelukkig prijzen dat ik Floris op mijn weg ontmoet heb.

*Freddy Dumortier*

**Culture**

For Floris Takens mathematics was embedded in a much larger scientific culture, in which Minnaert's *De Natuurkunde van 't Vrije Veld*, *The Feynman Lectures on Physics*, as well as *Gravitation* by Misner, Thorne and Wheeler, were never far from his desk. Apart from this,

he was also very interested in painting and music. He owned a large collection of paintings and regularly visited museums and exhibitions. For music he could be seen cycling through all weathers around the province of Groningen to performances in Leens, Feerwerd, Thesinge or in the Groningen Ooster-

poort.

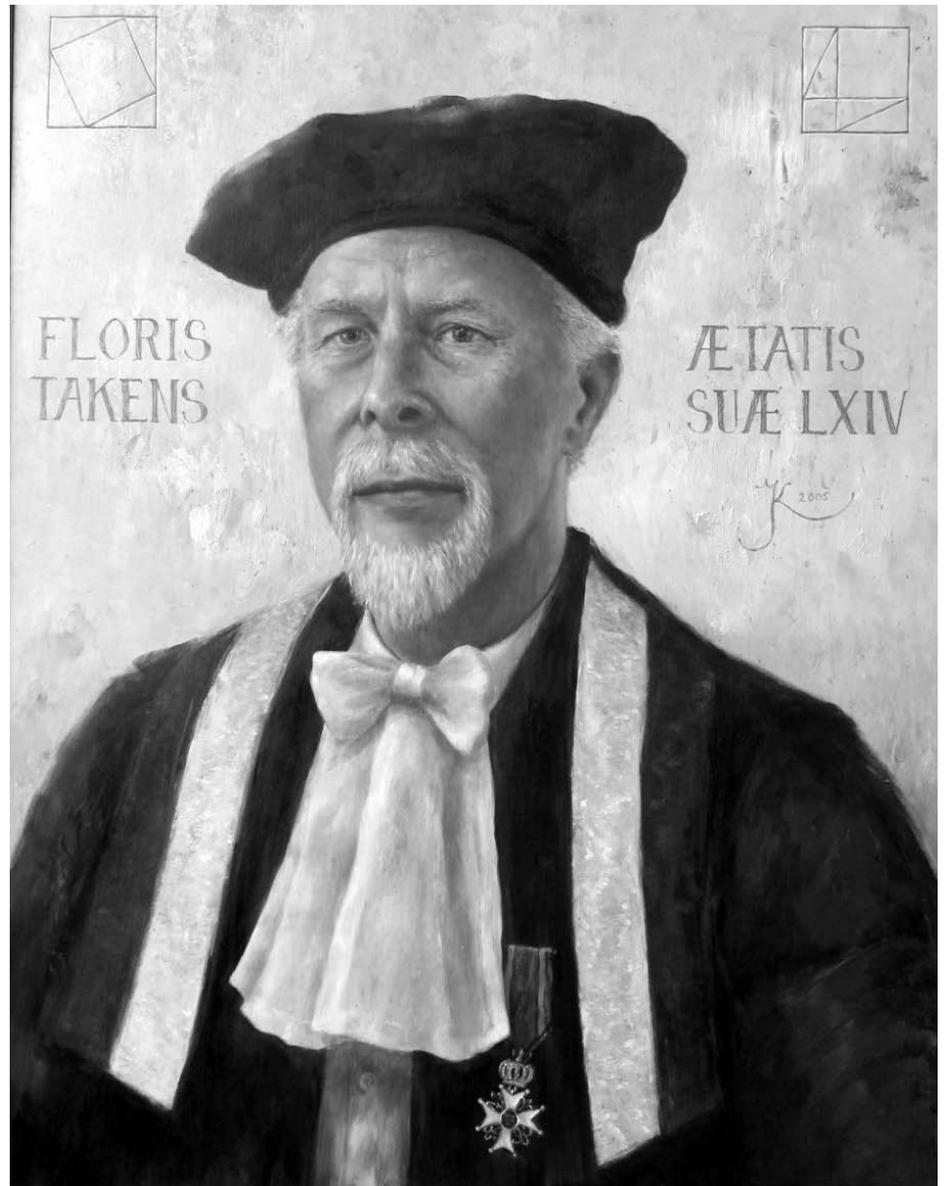
He was also actively involved in performing music. I cherish precious memories of numerous evenings in Bedum where Floris and I practiced and performed flute sonatas by Händel and Bach, now and then relaxing with Mozart's Andante for Flute in C or Gluck's

Dance of the Blessed Spirits. Floris played the traverso and I accompanied him on a virginal. An integral part of these evenings were the conversations afterwards, obviously with a good glass of wine. Apart from small talk and (local) politics, we also got around to matters of philosophy and theology. Floris had a clear affinity with the ideas of Spinoza and he was somewhat inclined to a form of pantheism.

Unfortunately Floris was witness to the undeniable decline of the scientific culture in our society over the past 40 years, as demonstrated by the decline in educational standards. Overly strict attention to market principles in research and education tends to replace true scientific quality and interest with a mere search for funding in the name of science. In a sense, the university itself has somewhat degenerated into a PhD thesis factory, often at the expense of scientific depth. The type of research that is driven by personal curiosity has too frequently been forced to abandon the field. In my opinion these developments sadly contributed to the fact that Floris Takens took early retirement at the age of 59.

A fitting anecdote in this regard is Floris's farewell lecture in 2001, where he discussed a report on the Monty Hall problem featured in the NRC newspaper some years previously. A series of articles and letters on the subject had been summarized by a journalist with the words: "Stop, stop, stop sending letters. The misunderstanding between common sense and the mathematicians is clearly unbridgeable." Floris observed this contempt for mathematics in a much broader sense, also within science. This tendency seems only to have grown stronger with time, and Floris acquiesced with a certain nostalgia.

To end on a more joyful note, let me say something about Floris's own free research which gave him much pleasure till the end of his life. This concerns the geometry of torus bundles, as mentioned before. In the very last



Portrait of Floris Takens painted by Jacqueline Kasemier in 2005

months of his life Floris produced an interesting sketch of a Morse Theory of monodromy and Chern classes, which will further occupy a number of us in the near future.

Finally I wish to express that a fasci-

nating human being has passed away with unmistakable grandeur. To Floris in person I say "Old soldier: it was an honour serving with you!"



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