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Interview N.G. de Bruijn and Hendrik Lenstra

Escher for the mathematician

In 1954, the International Congress of Mathematicians was held in Amsterdam. For the occasion, a special Escher exhibition was organized in the Stedelijk Museum. Professor N. G. de Bruijn (1918) was one of the people involved. About half a century later, Professor Hendrik Lenstra (1949) stopped at Escher's Print Gallery. Why is there a hole in the middle? Can it be filled, or is there a reason why this could not be done? He did get an answer to these questions, and since then he and his colleagues have travelled around the world giving lectures about it. Jeanine Daems visited De Bruijn and Lenstra and asked them about these episodes in their lives and about their ideas about Escher's work.

N.G. de Bruijn and Hendrik Lenstra both have been involved with the work of M. C. Escher. What is their opinion of his work?

Interview with N.G. De Bruijn

In what way did you come to know Escher's work? I knew Escher's work quite some time before 1954. He was not really well-known at the time. Still, every once in a while an article on his work would appear. Moreover, I happened to get a real Escher print in 1946.

In February 1945 (it was the last year of World War II) a Temporary Academy was set up in Eindhoven. The southern part of the Netherlands had been liberated, while the rest was still under German occupation. In the south, quite a few young people wanted to go to university but there was no place to study. In Eindhoven, with many scientists around, it was easy to find staff. I happened to work in Eindhoven with Philips from 1944 to 1946, which had saved me from forced labour in Germany, so I could give several courses in that academy.

At the dissolution of the academy in December 1945, Escher was asked to make this commemorative picture and it was presented to all the lecturers (but not to the students). I think about 80 copies were made.

There is a river in it that also appears in a famous picture of Escher's, with white on one side of it and black on the other, with reflection; you can see something of that in this one. But this picture is mainly about the owl sitting on the ruins and the broken chains, while there are still fires burning on the other side of the river.

When in 1954 the International Congress of Mathematicians took place in Amsterdam, an exhibition of Escher's work was attached. The idea came up in a discussion with my friend Seidel who was to be the leader of the entertainment activities around the congress. I was the secretary of the Program Committee, and in that capacity I was in close touch with the organizers of the congress. They appreciated the idea at once. Then I approached Escher; visiting him at his home (in Baarn) and I asked him how he felt about it. He liked it very much. Then I went to the managing director of the Rijksmuseum.

The Rijksmuseum? Yes, I went to the Rijksmuseum since several congress manifestations were to be held there. It was the kind of centre where everyone would go because it was world-famous. But the managing director replied: "No, we will not do this; it is modern art! You should be at the Stedelijk Museum for that". Well, at the time I already held the opinion that Escher's work was not art at all, not beautiful at all but very clever and very interesting. Escher was not a mathematician; and no artist. Escher was Escher. Nevertheless the Stedelijk Museum rather liked the plan.

The exhibition had to be opened, of course. I thought Freudenthal should do that because I considered him to be the leading mathematician in the Netherlands. But the congress committee — Koksma and others — said to me: "No, it was your idea, you should do it yourself". So I thought, okay, let's do it then. There I really overplayed my hand, having no idea what that meant — opening an exhibition. I expected that I would have to stand

The catalogue of the exhibition in 1954

on a crate in the exhibition room and say: "I now declare the exhibition to be opened."

How different it turned out. I was welcomed by the museum director and some other officials, such as the cultural deputy of the province of Noord-Holland. We had a cup of coffee and at some point they said: "Let us get started". I had no idea what was supposed to happen. First, one of the officials was to give a brief introduction and next I was to give the main speech. But I did not have a speech! We entered this *hall*! It was filled with people; there must have been over 150. But I had luck: this official gave a very long speech first - it took at least fifteen minutes. After that, I did not speak for more than five minutes, using the few notes I had made meanwhile. The effect was that there were many people who thought: "That first speaker took much too long but the second one did a good job!"

Some of my remarks found their way into the printed catalogue and Escher would quote them with pride later. They were not about the mathematical content of Escher's work but about the playfulness of it, the same kind of feeling we know from mathematics.

Mathematically, there was just one thing I had to report to Escher before the exhibition. I showed him pictures of the Klein model of hyperbolic space: the disk. He did not understand it at all, however, and since there were no photocopying machines yet my hint was lost. After the exhibition Coxeter approached Escher on this, and that made Escher start. Very nice, indeed. Previously, Escher had worked only with tessellations in connection with the 17 wallpaper groups. To Escher, who did not know any group theory, the 17 groups were just systems to make wallpaper. Those systems he knew: he had found most of them himself. Coxeter kept in touch with Escher in the years after the exhibition.

By the way, recent historical research has revealed even earlier contacts between Escher and George Pólya, who had written an article about the 17 symmetry groups in 1924 [Schattschneider, 2004].

Another interesting person whom Escher met at the congress was young Roger Penrose. His interest at that time was not tessellations but impossible figures, like stairs with water running up. As far as tessellations are concerned, I think it is a pity that Escher did not live long enough to see Penrose's famous nonperiodic tilings with imperfect fivefold symmetry in the early 1970s. Escher might have made nice tessellations out of those.

How did people react at the exhibition? Many people went there. It turned out to be one of the best attractions of the congress. In the Netherlands it did not have much influence, but the exhibition sparked off his fame in the international scientific world.

What do you think is the most beautiful in Escher's work? The most beautiful... well, you know, if it comes to beauty I am not really such an admirer of Escher.

You are not? No, you see, I understand how it was created, but Escher's artistic interpretation was somewhat wooden, out of sheer necessity. His human figures looked like wooden puppets. That is why regular artists did not want to call it art; what he did was not something they could recognize. But we like it, don't we?

Escher worked in black and white mainly. Had he used more colours, he might have produced more interesting symmetry issues. But that was difficult with the techniques he used. He was a great craftsman, of course, in a profession where a lot of patience and a lot of inspiration are required. Well, for mathematicians, Escher's work is definitely entertaining, but beautiful... What is beauty? I really do not know.

Interview with Hendrik Lenstra

Given De Bruijn's appreciation, what do you *hold of Escher as an artist?* Well, looking at Escher's work, there is no denying that he could not draw! He could not draw people, for instance. It is a bit like those Willink paintings: if in a painting by Willink a man walks down the street then it is not a walking person; it is a dummy in the shape of a walking person. Escher could not draw, and he admitted that himself as I read in his biography. But it is not done to criticize Escher. In my opinion it is an indisputable fact that Escher could not draw, and it shows in many of his prints.

How did you get the idea to look at the Print Gallery *from a mathematical point of view?* Well, being a mathematician, I see everything from a mathematical point of view! I got acquainted with Escher's work in highschool. In the early 1960s, Bruno Ernst started the highschool mathematics magazine *Pythagoras*. There he wrote articles about Escher's work, in a nice way, at the right level, but the most fascinating were the pictures. The *Print Gallery* was one of them. As a teenager I bought books on Escher but, to be honest, when I became a mathematics student, I considered it somewhat inferior: it was not 'real mathematics'.

With age one looks at things differently. As a mathematician you see certain things and you realise you do not understand them. I remember being on an airplane, reading about an Escher exhibition in one of these airline glossies. Here was the *Print Gallery* again and this time I looked at it from a completely different point of view. Clearly there was mathematics involved but I did not know *which* mathematics. In a way, the problem was my inability to formulate the problem. It is quite obvious what happens in the picture, and what Escher wanted to express. In

From the Foreword of the Catalogue of 1954

In view of the fact that Mr. Escher's work may be said to be a point of contact between art and mathematics, the Organizing Committee of the International Congress of Mathematicians 1954 (2-9 September) at Amsterdam took the initiative in inaugurating this exhibition.

Probably mathematicians will not only be interested in the geometrical motifs; the same playfulness, which constantly appears in mathematics in general and which to a great many mathematicians is the peculiar charm of their subject, will be a more important element.

It will give the members of the congress a great deal of pleasure to recognize their own ideas, interpreted by quite different means than those they are accustomed to using. *N.G. de Bruijn*

Image by Henry Segerman and Paul-Olivier Dehaye after an idea of Hendrik Lenstra

Hendrik Lenstra in front of the Memorial Church in Stanford

Escher literature it is called *circular expansion*. I tried an indirect approach: what if someone had this artistic idea and came to me during office hours — for the sake of argument, let us assume I had office hours — and asked me: "I want to express circular expansion. How should I do that? What does mathematics have to offer in this?". This is a way to ask the question, though not a very precise way. It is a question of the kind that applied mathematicians are more likely to be confronted with than pure mathematicians. Pure mathematicians have certain questions but these are mathematical questions already. The applied mathematician, by contrast, is confronted with the phenomena of the outside world, and the first step is to turn these into mathematics. To me that was the biggest problem: how to turn it into mathematics — how to express it in a mathematical way. Another thing that I wondered about, just to have a question to think about, was the hole in the middle of the *Print Gallery*. Some people say it is ugly, but that is not very relevant. My question was: is it necessary? There is mathematics involved and once the relevant mathematics is understood, obviously, one should be able to answer the question of whether the hole can be filled, or if there is a limit beyond which the filling would become a farce. But what does 'a farce' mean? I had not put that into words yet.

The drafts Escher made when working on

the *Print Gallery* had been published. Grids on these drafts show clearly what happens in the middle. I can imagine that Escher could see that, too. But I kept wondering about the mathematics; which functions were involved, for instance. Escher made four drafts, and every draft turned up four times smaller in the next draft. Evidently, there was a *Droste effect*. Eventually I arrived at a good question. In the drafts, I could see the picture was preserved under scaling by a factor $4^4 = 256$. I knew there had to be a repetition in the *Print Gallery* as well. This could be observed from the grid underlying the picture, but measurement of distances on the grid showed that the repetition in the grid was very different from that in the drafts. There was a rotation in the picture and this factor 256 was nowhere to be seen! There was a factor of about 20 and a rotation of 160 degrees. That was odd. So my question was: what are these numbers? And that was a precise question. I had an answer about five minutes later because it had become clear from the book by Bruno Ernst that the key concept was conformality. The transformation had to be conformal, which means that angles are preserved. Remember I asked the question: what would 'a farce' mean? Well, that means that the distortion is such that details are no longer recognizable. Bruno Ernst mentioned in his book that Escher was forced to bend the straight lines a bit because in that way the little squares in the grid would remain more like squares.

This made me realize that the drafts that I was looking at actually presented a quite well-known alternative way to describe elliptic curves over the complex numbers. The *Print Gallery* itself was drawn with a different period, and it was also an elliptic curve. And if the transformation between them is conformal then the elliptic curves have to be isomorphic. Well, I know everything about isomorphisms between elliptic curves, so I could answer my questions immediately. That was in January 2000. By now, the research question had turned into a nice topic to discuss over lunch.

I gave the first lecture about it in March 2000. Robbert Dijkgraaf was present and gave a new impulse to the work by asking: "Why don't you programme that?". Well, I did not because I am not a computer programmer.

The main people involved in the project were myself (I did not do much, though I had some money to spend on it) and Bart de Smit. And we needed someone for the programming work — that was Joost Batenburg, who was a student here in Leiden at the time.

As a project it really gained momentum when Sara Robinson, a science writer, proposed to write an article on it for the *New York Times*. We liked that, and looking ahead we realized that the article would generate a lot of questions and emails. We needed a place we could direct people to, so we made a website (escherdroste.math.leidenuniv.nl), which was launched the day the article appeared. Without Sara we would not have worked so hard on this.

Also, we had two artists working with us. The first was Hans Richter. We had to fill the hole in the middle. Because of the repetition, a considerable part of the hole can be filled using Escher's picture itself. But not all of it; a small part remained empty. In the straightened picture, this corresponded to an empty spiral. It was clear that it was possible to fill it; one would have to finish Escher's straight picture: continue some lines and so on. Richter did that. He was very enthusiastic about it; he understood the mathematics and he even found an inconsistency in Escher's picture: certain lines that should be parallel are not. Actually, Richter had to draw a bit himself: in our version of the *Print Gallery* there was a bit more space in the actual gallery and he drew his favourite Escher picture there: the Möbius strip with the ants. That was funny because this Möbius strip was made in 1963, whereas the *Print Gallery* is from 1956. Fantastic! Anachronisms. The impossible. That fits with Escher! The other artist was Jacqueline Hofstra: she 'coloured' the picture with shades of gray. She was very surprised at my enthusiasm about the anachronism. She had not expected mathematicians to like impossible things. People have strange ideas about mathematicians.

The project did, in fact, attract a good deal of publicity and we have given many lectures all over the world.

What do you think is interesting in Escher's work? What I find interesting are the things I do not understand. But I have to add that 'to understand' might not mean the same to different people. When I ask myself whether I understand something then I do not always find it easy to test whether I do. That is why I thought of questions like: when an artist comes into my office and asks this question, do I know what to tell him? And now I feel that I do. You have to check what mathematics this artist knows and on what level I can explain things to him. But I now know what to say. When one really understands a mathematical theory, one can explain it at any level.

But there are other pictures by Escher that I do not understand in this way. Sometimes it is just that I do not know the field, as is the case with hyperbolic geometry. I can imagine someone writing a book about that: 'Escher for mathematicians'. That could be a lovely book. It should not be aimed at high-school students but at mathematicians, and it should explain things in the way you would like to understand it yourself. I think a thorough examination of all of Escher's pictures would cover many areas of mathematics.

The ideas behind the *Print Gallery* could be applied by any artist. I am not really into doing that myself. It can be fun on occasion: we transformed Rembrandt's *Night Watch* and put that on the wall of our lunch room. It is a bit of a puzzle to get an artistically satisfactory result, but it is a kind of automatism. And we do not do automatisms in mathematics! When we understand something, it is no longer interesting.

References

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