Problemen

| Problem Section

This Problem Section is open to everyone. For each problem the most elegant correct solution will be rewarded with a 20 Euro book token. The judges reserve the right to withdraw the prize if none of the solutions is deemed worthy. The problems and results can also be found on the Problem Section website www.nieuwarchief.nl/ps.
Occasionally there will be a Star Problem, of which the editors do not know any solution. Whoever first sends in a correct solution within one year will receive a prize of 100 Euro.
Both suggestions for problems and solutions can be sent to uwc@nieuwarchief.nl or to the address given below in the left-hand corner; submission by email (in LaTeX) is preferred. When proposing a problem, please include a complete solution, relevant references, etc. Group contributions are welcome. Participants should repeat their name, address, university and year of study if applicable at the beginning of each problem/solution. If you discover a problem has already been solved in the literature, please let us know. The submission deadline for this edition is December 1, 2007.
The solutions of the previous edition will be published in the December issue.
The prizes for the Problem Section are sponsored by *Optiver Derivatives Trading*.

Problem A (Folklore)
Let a be an integer. Let $(x_n)_n$ be the sequence determined by $x_1 = a$ and $x_{n+1} = 2x_n^2 - 1$. Show that n and x_n are coprime for all n.

Problem B (Folklore)
Let G be a group with n elements and $S \subset G$ a non-empty subset. Show that the set $S^n := \{s_1 s_2 \cdots s_n | s_i \in S\}$ is a subgroup of G.

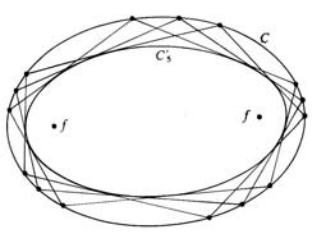
Problem C (Folklore)
(a) Given 2007 points in the plane such that no pair has distance strictly less than one, show that one can find a subset of 288 points in which no pair has distance strictly less than $\sqrt{3}$.
(b)* Supposedly the number 288 in problem C is not optimal. Find upper and lower bounds for the optimal value.

Problem 2006/3-C Consider a triangle ABC inscribed in an ellipse. For given A the other vertices can be adjusted to maximize the perimeter. Prove or disprove that this maximum perimeter is independent to the position of A on the ellipse.

Solution We received two submissions to this problem, one incorrect. The problem was solved by Jaap Spies, who remarked that it is a classical result, based on the Poncelet theorem, to be found in Darboux [1] Livre III, Chapitre III, part 176.
The statement of the problem follows from the Theorem of Chasles, which is given below. For a modern proof of this theorem, we refer to Berger [2], page 243.

THEOREM *Let C be an ellipse and $n \geq 3$ an integer. Among the convex polygons with n distinct vertices inscribed in C, there exist infinitely many with maximal perimeter. In fact, one vertex of such a maximal perimeter polygone (MPP) can be chosen arbitrary on C. Furthermore, the sides of all n-vertex MPP's are tangent to the same ellipse C'_n, homofocal with C.*
Jaap Spies remarks that for a treatment independent of Poncelet's Theorem see Lion [3].

[1] G. Darboux, *Principes de Géométrie analytique*, 1917 Gauthier-Villars, Paris. Available in facsimile: http://gallica.bnf.fr
[2] M. Berger, *Geometry II*, 1987, Springer Verlag, Berlin.
[3] George Lion, 'Variational Aspects of Poncelet's Theorem', *Geometricae Dedicata* **52**, 105–118, 1994.



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