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**Interview** Jean-Pierre Serre

# I hope we shall continue

**Drie april 2003 is de eerste Abelprijs toegekend aan Jean-Pierre Serre, emiritus hoogleraar aan het College de France, "voor de sleutelrol die hij heeft gespeeld bij het vormgeven van vele gebieden van de wiskunde waaronder topologie, algebraïsche meetkunde en getaltheorie." De Abelprijs is in 2001 ingesteld door de Noorse regering als wiskundige tegenhanger van de Zweedse Nobelprijs. Elk jaar zal een commissie van vijf Noorse wiskundigen een kandidaat aanwijzen. Aan de prijs is een bedrag van zes miljoen kronen (760.000 euro) verbonden. Onderstaand interview met Jean-Pierre Serre is afgenomen in Oslo op 2 juni gedurende de festiviteiten die de Abelprijs omringden.**

*First, we congratulate you on winning the first Abel Prize. You started your career with a thesis that centred on algebraic topology. This was then (at least in France) a very new discipline and not a major area. What made you choose this topic?* "I was participating in the Cartan Seminar, on Algebraic Topology. But Cartan did not suggest research topics to his students: they had to find one themselves; after that he would help them. This is what happened to me. I found that Leray's theory (about fibre spaces and their spectral sequence) could be applied to many more situations than was thought possible, and that such an extension could be used to compute homotopy groups."

*The methods and results that you created in your thesis revolutionised homotopy theory and shaped it in its modern look.* "They certainly opened up lots of possibilities. Before my thesis, homotopy groups of spheres were almost entirely terra incognita; one did not even know that they are finitely generated!

One interesting aspect of the method I introduced was its algebraic character. In particular, one could make 'local' computations, where the word 'local' here is taken as in number theory: relative to a given prime number."

*Is it true that one of the crucial points in this story was to identify something that looks like a fibre space without it being on the nose?* "Indeed, to apply Leray's theory I needed to construct fibre spaces which did not exist if one used the standard definition. Namely, for every space $X$, I needed a fibre space $E$ with base $X$ and with trivial homotopy (for instance contractible). But how to get such a space?

One night in 1950, on the train bringing me back from our summer vacation, I saw it in a flash: just take for $E$ the space of paths on $X$ (with fixed origin $a$), the projection $E \to X$ being the evaluation map: path → extremity of the path. The fibre is then the loop space of (X, a). I had no doubt: this was it! So much so that I even waked up my wife to tell her ... (Of course, I still had to show that $E \to X$ deserves to be called a *fibration*, and that Leray's theory applies to it. This was purely technical, but not completely easy.) It is strange that such a simple construction had so many consequences."

**Work Themes and Work Style**

*This story about your sudden observation is reminiscent of Poincaré's flash of insight when stepping into a tramway: this is told in Hadamard's booklet* The psychology of invention in the mathematical field. *Do you often rely on sudden inspiration or would you rather characterise your work style as systematic? Or is it a mixture?* "There are topics to which I come back from time to time ($l$-adic representations, for instance), but I do not do this in a really systematic way. I rather follow my nose. As for flashes, like the one Hadamard described, I have had only two or three in more than 50 years. They are wonderful ... but much too rare!"

*These flashes come after a long effort, I guess?* "I would not use the word 'effort' in that case. Maybe a lot of thinking. It is not the conscious part of the mind which does the job. This is very well explained in Littlewood's charming book *A Mathematician's Miscellany*."

*Most of your work, since the 'topology years', has been devoted to number theory and algebraic geometry.* "You see, I work in several apparently different topics, but in fact they are all related to each other. I do not feel that I am really changing. For instance, in number theory, group theory or algebraic geometry, I use ideas from topology, such as cohomology, sheaves and obstructions.

From that point of view, I especially enjoyed working on $l$-adic representations and modular forms: one needs number theory, algebraic geometry, Lie groups (both real and $l$-adic), $q$-expansions (combinatorics style) ... A wonderful mélange."

*Do you have a geometric or an algebraic intuition and way of thinking — or both?* "I would say algebraic, but I understand the geometric language better than the purely algebraic one: if I have to choose between a Lie group and a bi-algebra, I choose the Lie group! Still, I don't feel I am a true geometer, such as Bott, or Gromov.

I also like analysis, but I can't pretend to be a true analyst either. The true analyst knows at first sight what is 'large', 'small', 'probably small' and 'provably small' (not the same thing). I lack that intuitive feeling: I need to write down pedestrian estimates."

*You have had a long career and have worked on many different subjects. Which of your theories or results do you like most? Which are most important to you?* "A delicate question. Would you ask a mother which of her children she prefers?

All I can say is that some of my papers were very easy to write, and some others were truly difficult. In the first category, there is FAC (faisceaux algébriques cohérents). When I wrote it, I felt that I was merely copying a text which already existed; there was almost no effort on my part. In the 'difficult' category, I remember a paper on open sub groups of profinite groups, which gave me so much trouble that, until the very end, I was not sure whether I was proving the theorem or making a counter-example! Another difficult one was the paper dedicated to Manin where I made some very precise (and very daring) conjectures on *modular* Galois representations (mod p); this one was even painful; after I had finished it, I was so exhausted that I stopped publishing for several years.

On the pleasure side, I should mention a paper dedicated to Borel, on tensor products of group representations in characteristic $p$. I had been a group theory lover since my early twenties, and I had used groups a lot, and even proved a few theorems on them. But the theorem on tensor products, obtained when I was in my late sixties, was the first one I really enjoyed. I had the feeling that Group Theory, after a 40 years courtship, had consented to give me a kiss."

Martin Raussen and Christian Skau interviewing Jean-Pierre Serre

*You have been active in the mathematical frontline for more than 50 years. Hardy made the often quoted remark that 'Mathematics is a young man's game'. Isn't that wrong — aren't you a perfect counterexample?* "Not a perfect one: have you noticed that most of the quotations of the Abel Prize are relative to things I had done before I was 30?

What is true is that people of my generation (such as Atiyah, Borel, Bott, Shimura,...) keep working longer than our predecessors did (with a few remarkable exceptions such as Elie Cartan, Siegel, Zariski). I hope we shall continue."

**Relations to mathematical history**

*Since you've won the Abel Prize, we'd like to ask some questions with a background in Abel's time. The algebraic equations that Abel and Galois studied, coming from the transformation theory of elliptic functions, turned out to be very important much later for the arithmetic theory of elliptic curves. What are your comments on this remarkable fact, especially in connection with your own contribution to this theory?* "Yes, elliptic curves are very much in fashion (with good reasons, ranging from Langlands' program to cryptography). In the 60s and 70s I spent a lot of time studying their division points (also known as Tate modules) and their Galois groups. A very entertaining game: one has to combine information coming from several different sources: Hodge-Tate decompositions, tame inertia, Frobenius elements, finiteness theorems à la Siegel, ... I like that."

*Hermite once said that Abel had given mathematicians something to work on for the next 150 years. Do you agree?* "I dislike such grand statements as Hermite's. They imply that the person who speaks knows what will happen in the next century. This is hubris."

*In the introduction of one of his papers Abel writes that one should strive to give a problem a form such that it is always possible to solve it — something which he claims is always possible. And he goes on, saying that by presenting a problem in a well-chosen form the statement itself will contain the seeds of its solution.* "An optimistic point of view! Grothendieck would certainly share it. As for myself, I am afraid it applies only to algebraic questions, not to arithmetic ones. For instance, what would Abel have said about the Riemann hypothesis? That the form in which it is stated is not the good one?"

**The role of proofs**

*When you are doing mathematics, can you know that something is true even before you have the proof?* "Of course, this is very common. But one should distinguish between the genuine goal (say, the modularity of elliptic curves, in the case of Wiles), which one feels is surely true, and the auxiliary statements (lemmas, etc), which may well be untractable (as happened to Wiles in his first attempt) or even downright false (as happened similarly to Lafforgue)."

*Do proofs always have a value in themselves? What about, for example, of the proof of the four-colour theorem.* "We are entering a grey area: computer-aided proofs. They are not proofs in the standard sense that they can be checked by a line by line verification. They are especially unreliable when they claim to make a complete list of something or other.

I remember receiving in the 90s such a list for the subgroups of given index of some discrete group. The computer had found, let us say, 20 of them. I was familiar with these groups, and I easily found 'by hand' about 30 such. I wrote to the authors. They explained their mistake: they had made part of the computation in Japan, and another part in Germany, but they had forgotten to do some intermediate part ... Typical!

On the other hand, computer-aided proofs are often more convincing than many standard proofs based on diagrams which are claimed to commute, arrows which are supposed to be the same, and arguments which are left to the reader."

*What about the proof of the classification of the finite simple groups?* "You are pushing the right button. For years, I have been arguing with group theorists who claimed that the *Classification Theorem* was a theorem, that is, had been proved. It had indeed been announced as such in 1980 by Gorenstein, but it was found later that there was a gap (the classification of *quasi-thin* groups). Whenever I asked the specialists, they replied something like: "Oh no, it is not a gap, it is just something which has not been written, but there is an incomplete unpublished 800 pages manuscript on it".

For me, it was just the same as a "gap", and I could not understand why it was not acknowledged as such. Fortunately, Aschbacher and Smith have now written a long manuscript (more than 1200 pages) in order to fill in the gap. When this will have been checked by other experts, it will be the right moment to celebrate."

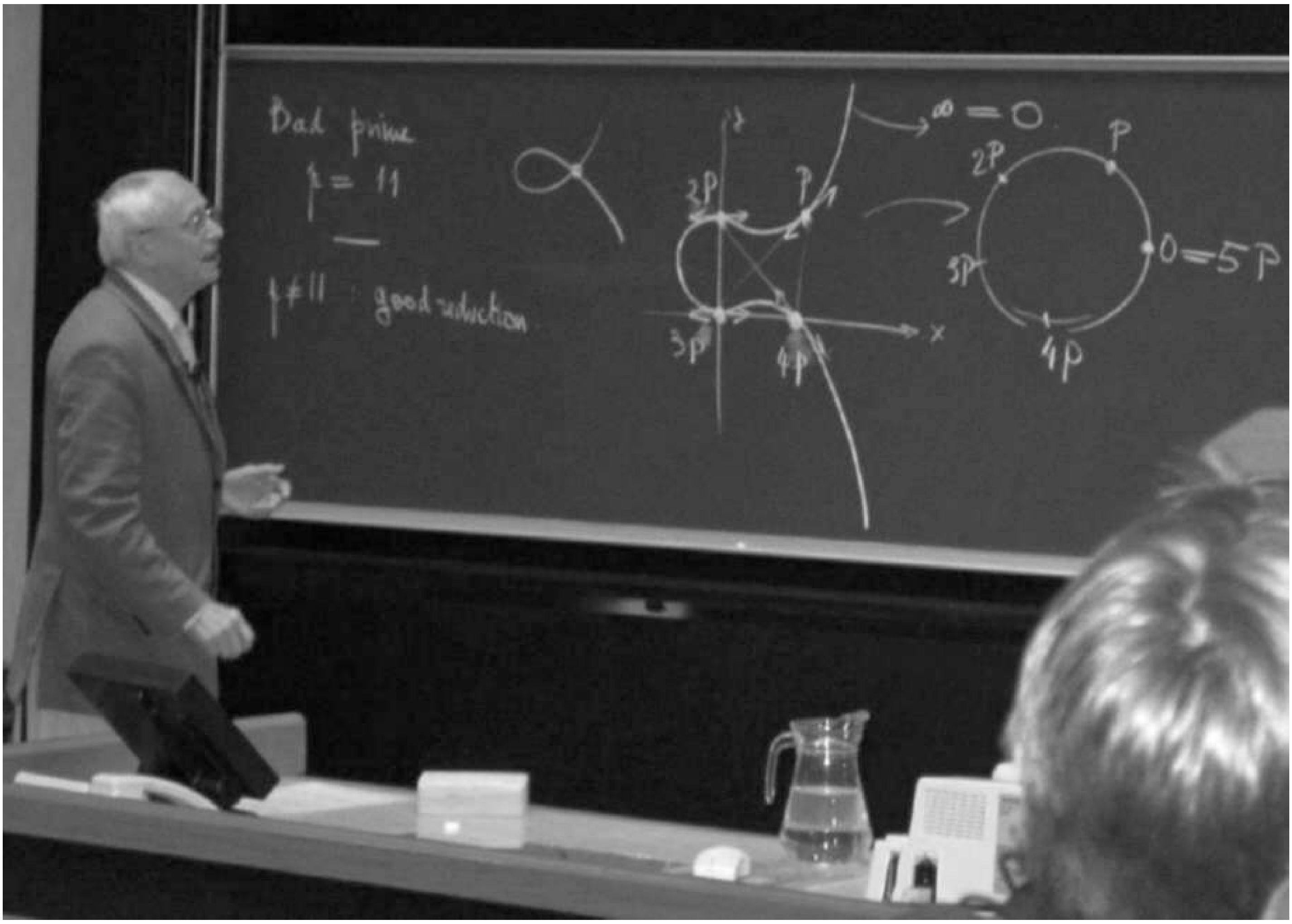
*But if a proof is 1200 pages long, what use is it?* "As a matter of fact, the total length of the proof of the classification is much more than 1200 pages; about 10 times more. But that is not surprising: the mere statement of the theorem is itself extremely long, since, in order to be useful, it has to include the detailed description, not only of the Chevalley groups, but also of the 26 sporadic groups.

It is a beautiful theorem. It has many very surprising applications. I don't think that using it raises a real problem for mathematicians in other fields: they just have to make clear what part of their proof depends on it."

**Important mathematical problems**

*Do you feel that there are core or mainstream areas in mathematics — are some topics more important than others?* "A delicate question. Clearly, there are branches of mathematics which are less important; those where people just play around with a few axioms and their logical dependences. But it is not possible to be dogmatic about this. Sometimes, a neglected area becomes interesting, and develops new connections with other branches of mathematics.

On the other hand, there are questions which are clearly central for our understanding of the mathematical world: the Riemann hypothesis and the Langlands program are two obvious cases. There is also the Poincaré




As a part of the ceremonies, Jean-Pierre Serre delivered a lecture at Universitetet i Oslo.

conjecture — which may well stop being a conjecture, thanks to Perelman!”

*Do you have more information, or a hunch, about the correctness of the proof?* “Hunch? Who cares about hunches? Information? Not really, but I have heard that people at IHES and MIT are very excited about this sketch of proof. An interesting aspect of Perelman’s method is that it uses analysis, for what is a purely topological problem. Very satisfying.”

*We have already moved a little into the future with our discussion of the Poincaré conjecture. Which important mathematical problems would you like to see solved in the near future? And do you agree with the primary importance of the Clay Millennium Prize Problems?* “Ah, the million dollars Clay problems! A strange idea: giving so much money for one problem . . . but how can I criticise it, just after having received the Abel prize? Still, I feel there is some risk involved, namely that people would shy from discussing their partial results, as already happened ten years ago with Fermat’s theorem.

As for the choice of questions made by the Clay Institute, I feel it is very good. The Riemann hypothesis and the Birch & Swinnerton-Dyer conjecture are rightly there. The Hodge conjecture, too; but for a different reason: it is not clear at all whether the answer will be yes or no; what will be very important will be to decide which (I am hoping, of course, that it will not turn out to be undecidable . . .). The $P = NP$ question belongs to the same category as Hodge, except that there would be many more applications if the answer turned out to be “yes”.”

*Can you think of any other problems of the same stature?* “I already told you that the Langlands program is one of the major questions in mathematics nowadays. It was probably not included in the Clay list because it is very hard to formulate with the required precision.”

*Besides your scientific merits, you are also known as a master expositor, as we witnessed during your lecture today.* “Thanks. I come from the South of France, where people like to speak; not only with their mouth, but with their hands, and in my case with a piece of chalk.

When I have understood something, I have the feeling that anybody else can understand it too, and it gives me great pleasure to explain it to other mathematicians, be they students or colleagues.

Another side of the coin is that wrong statements make me almost physically sick. I can’t bear them. When I hear one in a lecture I usually interrupt the speaker, and when I find one in a preprint, a paper or in a book I write to the author (or, if the author happens to be myself, I make a note in view of a next edition). I am not sure this habit of mine has made me very popular among lecturers and authors.”

## Accessibility and importance of mathematics

*Mathematics witnesses an explosion of subjects and disciplines, making it difficult to master even the minor disciplines. On the other hand — as you demonstrated today in your lecture — it is very important that disciplines cross-fertilise each other. How can young mathematicians, in particular, cope with this explosion of knowledge and come up with something new?* “Oh yes, I have already been asked that question in my Singapore interview, reproduced by *Intelligencer*.[1] My answer is that, when one is truly interested in a specific question, there is usually very little in the existing literature which is relevant. This means you are on your own.

As for the feeling of ‘explosion’ of mathematics, I am convinced that Abel felt the same way when he started working, after Euler, Lagrange, Legendre and Gauss. But he found new questions and new solutions. It has been the same ever since. There is no need to worry.”

*Another current problem is that many young and talented people — and also public opinion leaders — don’t think that mathematics is very exciting.* “Yes. Sadly enough, there are many such examples.

A few years ago, there was even a French minister of Research who was quoted as saying that mathematicians are not useful any more, since now it is enough to know how to punch a key on a computer. (He probably believed that keys and computer programs grow on trees . . .)

Still, I am optimistic about young people discovering, and being attracted by, mathematics. One good aspect of the Abel festivities is the Norwegian Abel competitions, for high school students.”

## Sports and literature

*Could you tell us about your interests besides mathematics?* “Sports! More precisely: skiing, ping-pong, and rock climbing. I was never really good at any of them (for example, when I skied, I did not know how to slalom, so that I would rather go ‘schuss’ than trying to turn); but I enjoyed them a lot. As luck has it, a consequence of old age is that my knees are not working any more (one of them is even replaced by a metal-plastic contraption), so that I had to stop doing any sport. The only type of rock-climbing I can do now is a vicarious one: taking friends to Fontainebleau and coaxing them into climbing the rocks I would have done ten years ago. It is still fun; but much less so than the real thing. Other interests:

- movies (*Pulp Fiction* is one of my favourites — I am also a fan of Altman, Truffaut, Rohmer, the Coen brothers . . .);
- chess;
- books (of all kinds, from Giono to Böll and to Kawabata, including fairy tales and the *Harry Potter* series).”

*Prof. Serre, thank you for this interview on behalf of the Danish and the Norwegian Mathematical Societies.*

This interview appeared originally in 2003 in the Newsletters of the Norwegian, the Danish and the European Mathematical Societies.

## References

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3 Pilar Bayer, ’Jean-Pierre Serre, medalla Fields’, *La Gaceta* **4** (1) (2001), p. 211–247.