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Boekbespreking De rekenmeester

Dichtung und Wahrheit

In het Nieuw Archief van september 2000 geeft Nicky Hekster een bespreking van het boek 'De rekenmeester' van Dieter Jörgensen. Daarbij is hij ingegaan op de literaire kwaliteiten van het boek. Friedrich Katscher, wetenschapshistoricus en in de Nederlandse wiskundige gemeenschap bekend als auteur over het werk van Ludolph van Ceulen, besteedt nu aandacht aan de historische aspecten van de roman.

In the history of mathematics there is no other epochal book like the *Quesiti et Inventioni diverse* (Diverse questions or problems and inventions) written by Nicolo Tartaglia in 1546, and containing the first printed solution of the cubic equations without the quadratic term. Its peculiarity is that the accustomed calculations, numbers, symbols, and geometrical figures as in other mathematical books are here inserted in lively dialogues and correspondences which tell who was the questioner, the good or bad relationship the author had with him, and all the circumstances. Even the bitter youth of Tartaglia is described, and all the details of the famous controversy between Tartaglia and Hieronimo Cardano can be followed although from the one-sided and biased point of view of the author. The eventful book is almost a sketch for a film-script, and, in fact, a German has written a novel about the life of *De Rekenmeester* (German: Der Rechenmeister).[1]

It is unusual that a journal publishes two reviews of the same book. But the first boekbespreking[2] of this novel suffers from the great credulity of the reviewer. N.S. Hekster knows the historical details quite well as his timetable (Tijdslijn) proves.[3] Therefore, it is regrettable that he believed all the inventions and fancies of Jörgensen. The German engineer who, according to the dust cover, publishes articles about problems of applied mathematics in the trade press writes a few details in his book with great precision — that means, he has read some original passages of Tartaglia and Cardano — but most of the book arose from his imagination.

Truth and imagination

It is not for a mathematical journal to judge the literary quality of a novel but only the correctness of mathematical and historical text passages. The author of fiction has full poetic freedom to contrive a plot but if someone should write, for example, a novel about Albert Einstein he would also be measured by how far he strays from the factual biography of the great scientist.

There are three Italian books where everything can be found about the life and works of Nicolo Tartaglia (1499/1500–1557):

- The proceedings of the conference held in May 1959 on the occasion of the 400th anniversary of Tartaglia's death in his birth town[4] ;
- A facsimile edition of the second edition of the *Quesiti* of 1554[5] ; and
- A facsimile edition of the 12 mathematical pamphlets, six from each one, exchanged between Tartaglia and Cardano's disciple and collaborator Lodovico Ferrari in 1547 and 1548[6] .

All three books were published by the Ateneo di Scienze, Lettere ed Arti da Brescia, the academy of this North Italian town. They are illustrated with many plates.

Arnaldo Masotti (1902–1989), an outstanding Tartaglia expert, had gathered with meticulous thoroughness and precision all the information that exists in books and documents about the great renaissance mathematician. In his introduction to the two facsimile editions there is a chapter describing the work, an index of persons, an index of dates, and observations about archaic expressions. It seems impossible that any new relevant facts could be found about Tartaglia not mentioned in these three volumes. Therefore, they can be used to see how far Jörgensen wrote facts and fictions.

The novel is based on two assumptions: Firstly, that Tartaglia stuttered and had difficulties in speaking; and secondly, that he was single. The Italian word Tartaglia means stutterer. At the end of the Sixth Book of the *Quesiti* the author describes why he adopted this name. When the French army on 19 and 20 February 1512 assaulted Brescia, and massacred the population, the 12 year old boy received five sword-strokes on his head by a soldier in the cathedral of Brescia. One across his mouth and his teeth split his upper and lower jaws and his teeth. "... there was a time when I was not able to articulate the words well, but always stuttered in speaking because of this injury across the mouth and the (not yet well fixed) teeth, and that's why the boys of my age with whom I talked gave me the nickname Tartaglia. And because this surname remained me a long time it seemed to me to be a good memory of my misfortune to call me Nicolo Tartaglia."

According to this report — "there was a time..."[7] — it seems that Tartaglia stuttered only for some time (probably until his teeth were fixed), and not for his whole life. In the *Quesiti* (*Quesito* — question — *XXII* and *XXXII*) Tartaglia reported that he held lectures in the Venetian church of Santi Giovanni e Paolo (Zanipolo) in 1536 about Euclid's *Elements* and in 1539 about the science of weights (static) and artillery shots (ballistics). He could not have lectured if he had stuttered as described by Jörgensen. By the way, on the dust cover of *De Rekenmeester* it is stated that Tartaglia had to witness how his mother was violated in the cathedral. This is untrue.

In Verona there is a Latin document of 1529 in which a Nicolaus brixiensis magister Abbachi (Nicolo from Brixia = Brescia, Master of

Abacus, that is arithmetics), certainly Tartaglia, aged 30 years, is listed with his much older wife Dominica, 44, his (step)daughter Benevenuta, 18, his daughter Margarita, 2, and Benevenuta's daughter Anna, 6 months. In *La Seconda Parte del General Trattato di Numeri et Misure* (The Second Part of the General Treatise of Numbers and Measures, Venice 1556) by Nicolo Tartaglia on folio 41 verso he mentions his family in 1548. That means in the years depicted in the novel he was married, he had not to go to a prostitute, and there was no sexual relationship with the beautiful Jewess Sara. Tartaglia must have lost his family later because neither his wife nor his daughters are mentioned in his last will of 1557.

Eternal secrecy

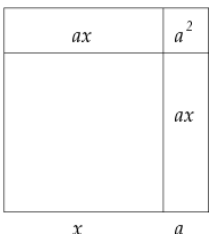
Until the downfall of the Republic of Venice in 1797 according to the "mos Venetus" (Venetian use) the year did not begin officially on the 1st of January but, as in the old Roman calendar before Julius Caesar, on the 1st of March. That means January and February belonged to the previous year according to our calculation of years. In the novel Tartaglia arrives in Venice coming from Verona on 12 February 1534 of Venetian numbering, that is, on 12 February 1535 of our years. But in the *Quesiti* the title of *Quesito XIX* says: "Asked by magnifico (Venetian nobleman) mister Zuanbattista Memo in the year when I came to reside in Venice which was 1534."[8] Tartaglia always used our chronology.

In *Quesito XXV* of 10 December 1536 Tartaglia writes that challenged by the arithmetician Antoniomaria Fior who set him 30 problems — only cubic equations of the type $x^3+px=q$ — he discovered the solution of this equation called 'cosa e cubo equal à numero' (unknown and cube equal to number) "last year, that is 1535, on the day 12 of February (it is true that in Venice it came to be 1534)."[9] That means that the day of Tartaglia's arrival in Venice in the novel is the day when in reality Tartaglia found the solution of the cubic equation which Jörgensen puts into a chapter titled "17 January 1535 of Venetian time counting" — that is 17 January 1536 in our numbering, about eleven months too late.

Implored by Cardano ("*Quesito XXXIIII*, asked personally by Mister Hieronimo Cardano in Milan in his house on 25 March 1539") who vowed eternal secrecy, Tartaglia confided in him the secret of the solution of the cubic equation without the quadratic term in a poem in which he described the method: Take two numbers the difference of which is the "numero", the number q of the equation (modern) $x^3+px=q$. That is $u-v=q$. Their product should be equal to the cube of the third of the "pure unknowns" p. That is: $uv=(p/3)^3$. That means, you have two equations to find u and v. Solve them. The solution of the cubic equation is $x=\sqrt[3]{u}-\sqrt[3]{v}$.

Al-Hwarizmi

How did Tartaglia find this solution? He gives no hint. But he knew Euclid's *Elements* well. He lectured about them, and he translated them into Italian in a book published in 1543. Book X of the *Elements* treats binomials of irrational quantities which today would be written



The method of al-Khowarizmi. The gnomon consists of the two rectangles $2ax$ and the square a^2.

foto: Kunsthistorisches Museum, Wenen

Berthel Beham, *Bildnis eines Mannes*. Compare this picture with the cover of the book (see review NAW september 2000)

as $a+\sqrt{b}$, $\sqrt{a}+b$, $\sqrt{a}+\sqrt{b}$, $a-\sqrt{b}$, $\sqrt{a}-b$, and $\sqrt{a}-\sqrt{b}$. The solution of quadratic equations contains square roots. It could be assumed that the solution of cubic equations contains cube roots. Tartaglia probably experimented with the different binomials of cube roots. When he inserted (modern) $x=\sqrt[3]{u}-\sqrt[3]{v}$ into the equation $x^3+px=q$ he obtained

$$u-v-3\sqrt[3]{u^2v}+3\sqrt[3]{uv^2}+p(\sqrt[3]{u}-\sqrt[3]{v})=q.$$

An important step was to write the third and fourth term as $-3\sqrt[3]{uv}(\sqrt[3]{u}-\sqrt[3]{v})$ to get

$$u-v-3\sqrt[3]{uv}(\sqrt[3]{u}-\sqrt[3]{v})+p(\sqrt[3]{u}-\sqrt[3]{v})=q.$$

It is easy to see that you can equate $u-v$ with q, and $3\sqrt[3]{uv}$ with p which gives $uv=(p/3)^3$. Thus you have two equations to calculate u and v. The solution x is then $\sqrt[3]{u}-\sqrt[3]{v}$. It is admirable how Tartaglia found this without the modern symbolism, and mainly geometrically.

Abu Abdallah Muhammad ibn (ben) Musa al-Khowarizmi (his name appears in many different spellings; Jörgensen writes al-Hwarizmi) who worked in the library of caliph al-Mamun who reigned from 813 to 833 in Baghdad solved the quadratic equation (modern) $x^2+10x=39$ with the help of a 'gnomon'. That is the Greek term used by the pythagoreans for the remaining area that is obtained by cutting out a smaller square in one corner of a bigger square, and consisting of two rectangles and a square at the opposite corner. When the side of the square that is cut out is x, and the side of the whole square is $x+a$ the area of the gnomon is $(x+a)^2-x^2=2ax$ (the two rectangles) + a^2 (the

Nicolo Tartaglia always wore a full beard and a barret to hide the disfiguring scars from the serious wounds he received as a boy on his head and across his mouth and jaws through five sword-strokes of a French soldier. (Portrait at the age of about 45 years on the title page of the *Quesiti*.)

opposite square). Muhammad ben Musa equated the areas $10x$ and $2ax$. That leads to $a = 5$, the side of the opposite square. The Arab mathematician added the area of this square $a^2 = 25$ on both sides of the equation: $x^2 + 10x + 25 = 39 + 25 = 64$. That is the area of the whole square. Its side is $\sqrt{64} = 8$. $x = 8 - 5 = 3$.

In the novel Tartaglia studies this method of al-Hwarizmi, and, by using a solid three-dimensional gnomon instead of the plane two-dimensional gnomon of the Arab mathematician, he finds the solution of the cubic equation $x^3 + px = q$. That seems plausible but in fact al-Hwarizmi's method does not work in three dimensions because in this case you would have to equate six solids, six square prisms, $3x^2a + 3xa^2$, with a plane rectangle, px. Cardano knew al-Khowarizmi (although not by this name), and he begins his *Ars magna* (The great art, 1545)[10] with the sentence "This art (the algebra) originated with Mahomet the son of Moses the Arab." He also describes the method of al-Khowarizmi for obtaining the solution of the quadratic equation without using his name and the term 'gnomon'. But it seems that Tartaglia did not know the existence of al-Hwarizmi. He does not mention him in the *Quesiti* nor in the *Cartelli*.

A beard to disguise his scar

Eight persons Tartaglia meets in the novel, among them Fior, Cardano, Ferrari and the Duke of Urbino, Francesco Maria delle Rovere, are historical. But the many other Venetians mentioned in the book are all fictitious. The refusal of the 'privilegio', the permission to print Tartaglia's book on ballistics *La Nova Scientia* (New Science) of 1537, is an invention of Jörgensen. The page in Luca Pacioli's *Summa de Arithmetica Geometria Proportioni e Proportionalita* (1494) where he writes that the solution of the cubic equations is impossible is given twice in the novel as folio 216. But in reality it is folio 149 recto. The title of Cardano's book published in 1539 is not *Practica Arithmetica generalis* but *Practica Arithmetice, & Mensurandi Singularis*.

In his review Hekster writes: "In his later life Fontana (Tartaglia) habitually wore a (full) beard in order to disguise his scars." Tartaglia himself wrote that he would appear a monster without the beard.[11] But a few lines later Hekster says: "The dust cover of the book shows an approximately thirty year old beardless Tartaglia on a painting by the Nuremberger Barthel Beham from 1529. Clearly there is a scar to see on his right cheek." The painter Barthel Beham (1502–1540) lived in 1529 in Munich. How should he paint an unknown arithmetician in Verona? The painting is in Vienna in the Kunsthistorische Museum (Museum of Fine Arts).[12] It is called 'Portrait of a Man' (Bildnis eines Mannes), not even 'Portrait of a *reckoning* Man'. The face has no scar at all on the right cheek. It is spotless. That means, that the scar has been added by the artist who made the dust cover of the book. But it does not go across the mouth as it should.

Jörgensen has gained merit by making public a formerly largely unknown famous Italian mathematician of the 16th century. For a non-mathematical reader it does not matter whether the details of the novel are correct or not. If he finds the book worth reading he learns at least something about the circumstances of the life of a Venetian mathematician in the Renaissance. But readers interested in the history of mathematics have to be warned: The greatest part of *De rekenmeester*, praised on the dust cover as a fascinating insight into the history of science, is not reliable fact but pure fiction.

De rekenmeester, Dieter Jörgensen, 336 pages, price € 19,50, BZZTôH, Den Haag, 2001, ISBN 9-05501-722-1. German original: ***Der Rechenmeister***, 398 pages, Rütten & Loening, Berlin, 1999, sold out, ISBN 3-352-00555-9, ***Der Rechenmeister***, 400 pages, Aufbau Verlag, 1999, € 9.50, ISBN 3-74661-704-9.

Notes

1 Rechenmeister, literally reckoning-master, means arithmetician in English.

2 N.S. Hekster, 'De rekenmeester', *Nieuw Archief voor Wiskunde* 5/1 nr. 3, september 2000.

3 There are only three mistakes: Tartaglia was already in Venice in 1534, not 1535, the mathematical contest Tartaglia-Fior was in 1535, not 1536, and the visit of Cardano and Ferrari to Bologna when they saw the solution of the cubic equation by Scipione dal Ferro was in 1542, not 1543.

4 *Atti del Convegno* di Storia delle Matematiche 30–31 Maggio 1959 (Editor: Arnaldo Masotti), Brescia 1962.

5 Niccolò Tartaglia, *Quesiti et Inventioni diverse*. Introduction by Arnaldo Masotti, Brescia 1959.

6 Lodovico Ferrari e Niccolò Tartaglia, *Cartelli di Sfida matematica*. Introduction by Arnaldo Masotti, Brescia 1974.

7 The Italian original: "... stetti un tempo, che io non poteva ben proferire parole, ma sempre balbutava nel parlare..."

8 The Italian original: "... l'anno ch'io veni ad habitare in Venetia, che fu .1534."

9 The Italian original: "... questo fu l'anno passato, cioe del .1535. adi .12. di .Febraro (vero è che in Venetia veneva esser del .1534.)" Numbers were always written within two dots.

10 There is an English translation of the *Ars magna*: Girolamo Cardano, *Ars magna or the Rules of Algebra*, translated and edited by T. Richard Witmer. Dover Publications, New York 1993.

11 The Italian original: "... se la barba non me le occultasse, io pareria un mostro..."

12 Eckzimmer — corner-room — 17; Inventory number 783.