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Recreational mathematics

# An algorithm for the generation of perfect squared rectangles of arbitrary dimension

A *perfect squared rectangle* (PRS) is a rectangle composed of squares of all different sizes. Many examples are given in the literature, for example in [1], [2] and [3].

In the following, an algorithm will be presented for the construction of a new PSR, starting with an initial PSR.

**Step 1.** Take an existing PSR with sides $a$ and $b$.

**Step 2.** If $a - b \neq 0 \bmod 3$ enlarge the PSR to dimensions $a := 3a$ and $b := 3b$, which means that all sides of the embedded squares are being multiplied by 3 as well.

**Step 3.** Construct a new PSR as follows, adding size squares $p$ through $u$ (see figure 1) with dimensions:

$$\begin{aligned} p &= (6a + 6b)/3, \\ q &= (7a + 5b)/3, \\ r &= (4a + 5b)/3, \\ s &= (4a + 2b)/3, \\ t &= (\ a + 2b)/3, \\ u &= (5a + 4b)/3. \end{aligned}$$

The new rectangle has dimensions $p + u = (11a + 10b)/3$ and $p + q = (13a + 11b)/3$.

The so generated figures is a PSR because:
1. Squares $p$ through $u$ have integer sides since all numerators in the expressions for these squares are triples.
2. The figure clearly is a rectangle as can be checked by summing the various horizontal and vertical dissections. Moreover, the new rectangle is not a square because $p + u \neq p + q$. As such, it can be used again for the generation of a larger PSR using the same method.
3. As is easily verified, squares $p$ through $u$ are all different and bigger than the squares contained within the initial PSR. In fact, we have the ordering:

$$\begin{aligned} &\text{if } b < a, \text{ then } b < t < a < s < r < u < p < q, \\ &\text{if } b > a, \text{ then } a < t < b < s < u < r < q < p. \end{aligned}$$

**Remark.** A different PSR is obtained whether starting with $b > a$ or $b < a$. The latter case gives a bigger PSR because so are sides $p + u$ and $p + q$.

**Example.** Take as initial PSR the well known PSR of dimensions $32 \cdot 33$, filled with squares of size 18, 15, 7, 8, 14, 4, 10, 1, 9, where the dissection is from upper-left to lower-right in the ordering indicated. Because the sides are 32 and 33, we have $a - b \neq 0 \bmod 3$. Following step 2 above, the starting dimensions are $a = 3 \cdot 32$ and $b = 3 \cdot 33$. The new PSQ consists of the original squares (multiplied by 3): 54, 45, 21, 24, 42, 12, 30, 3, 27 and squares:

$$p = 390,\ q = 389,\ r = 293,\ s = 194,\ t = 98,\ u = 292,$$

leading to a PSR of dimensions $p + u = 682$ and $p + q = 779$.

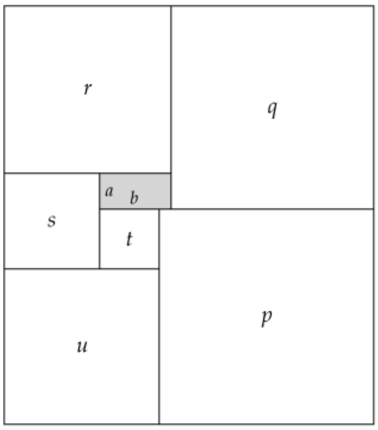


Figure 1

**References**

1 A.J.W. Duijvestijn, *A lowest order simple perfect* $2 \times 1$ *squared rectangle*. Journal of Combin. Th. Ser. B **26**, 372–374, 1979.

2 H. Reichert and H. Toepken, H. Jahresber. Deutsche Math. Verein. **50**, 1940.

3 H. Steinhaus, *Mathematical Snapshots*, 3rd Ed. New York, Dover, 1999.