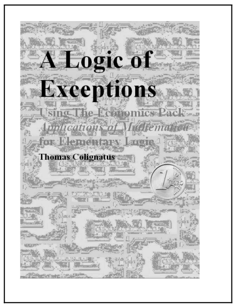
Boekbesprekingen

Book Reviews



Thomas Colignatus

A Logic of Exceptions: Using the Economics Pack Applications of Mathematica for Elementary Logic

Scheveningen: Thomas Cool Consultancy & Econometrics, 2007
252 p., pdf-version freely available at http://www.dataweb.nl/~cool/Papers/ALOE/Index.html, ISBN 978-90-804774-4-5

This is not really a book review. But it is a warm recommendation to download and read a free and self-published book on self-reference called *A Logic of Exceptions* (ALOE 1981 / 2007) by Thomas Colignatus — the science name of Thomas Cool (b. Jakarta 1954).

The author is an econometrician who many years ago lost his job at the Central Planning Bureau (CPB, the Netherlands government econometric planning agency) when his employer suppressed his research on unemployment. Colignatus is an original thinker and where I come from (England) we cherish excentrics, but in the polder-landscape of the Netherlands even mild unconventional thinking may get you into trouble. Without wanting to start a discussion about excentricity, or misrepresent the author or his Dutch environment, let me merely state that I think that the author is excentric and simultaneously I mean that as a compliment to him. Moreover, I simultaneously think that he is a decent scientist who asks (interesting) questions and writes (interesting) analyses of the answers which he finds. For me there is a positive correlation between these various opinions. These remarks are not irrelevant to the present book as I will explain later.

The idea of ALOE is that three-valued logic is the way to solve all the paradoxes in logic concerning self-reference. In particular, the author claims to turn Gödel's self-referential statement that it itself cannot be proved, into just another variant of the famous liar paradox. How does three-valued logic come into this? Well, by the idea of letting the third truth value not stand for 'the truth of this statement is undetermined' but for 'this statement is nonsense'; i.e., indeterminate instead of undetermined. I do not know if a three-valued logic having such an interpretation has been studied before, but do notice that the rules of such a logic ought to differ from the rules for a more conventional three-valued logic. In two-valued logic 'A implies B' is equivalent to 'not-A or B' and conventional three-valued logic retains this, so an implication is undetermined if A or B is undetermined. Now, suppose that we represent 'true', 'false' and the new third truth-value 'nonsense' by the numbers 1, 0 and 1/2. In Colignatus's calculus of nonsense, implication remains two-valued, and can be conveniently and faithfully represented numerically with 'less than or equal'. Equivalence remains two-valued as well, and is represented numerically with equality. In particular, 'nonsense implies nonsense' is true (the author's framework), while 'indeterminate implies indeterminate' is indeterminate (conventional three-valued logic). Colignatus holds that his approach avoids the liar paradox that arises in conventional three-valued logic. The 'trick' is not having levels in language or a theory of types but having cumulations in nonsense. The statement of the Liar (that he always lies) is not undetermined, but nonsense (in principle indeterminate).

The three-valued logic and its rules are added by Colignatus as

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part of a grander plan (to him Gödel is only secondary) to defuse the danger of self-reference and solve the liar paradox, and its many variants. He wants to add an axiom to ordinary logic which actually legitimizes self-reference in the following way: an axiom which states that if a certain proposition can be deduced from the axioms, then the system knows that the proposition has a proof. The system is axiomatically allowed to talk about itself. This thus uses the distinction of selfreference of sentences vs systems. Adding that axiom but keeping conventional logic would lead to an inconsistent system, precisely because of the liar paradox. Also the Gödel statement is both true and not true. Conventional three-valued logic gives no escape. However, with Colignatus's three-valued logic, both the Liar statement and the Gödel statement become merely nonsense. (Recall that in the usual world, assuming that ordinary number theory is consistent, the Gödel statement is a true statement of number theory which does not have a proof within the conventional formalism of number theory. Consequently, one can even add its negation to the usual axioms and still have a consistent system.)

Many readers may feel uncomfortable here. Gödel's work is one of the crowning glories of 20th century mathematics. It has had enormous impact in mathematics, not just in some academic branch of mathematical logic, but it has changed the way that mathematicians think about mathematics. It is one route to the theory of nonstandard analysis. It is a cherished jewel, and now a mere econometrician says it is just based upon nonsense? It was all a big mistake?

This would be to misunderstand Colignatus's intentions. In fact, it is falling into the trap of confusing different layers of discourse on the Gödel theorem. On the one hand, Colignatus does not show that Gödel's proof was wrong, but he does show that Gödel himself drew untrue (or at least, misleading) conclusions about it. On the other hand, logic is something which we use not just inside mathematics but also in science, in law, in life. Why should we necessarily use the same formal logic inside mathematics and inside, say, government and bureaucracy? Or in empirical science? I think it makes sense to develop a logic of exceptions, a logic which does not assume that every statement which can be made in a certain realm is either true or false. It is certainly very easy to talk nonsense when talking about the real world. Finally, the logical paradoxes arise from mathematical considerations on selfreference, and it is certainly pretty easy to talk nonsense about selfreference, so solution approaches developed with one purpose in mind may be useful in others.

Changing some of the ingredients of the usual axiomatics of the foundations of mathematics, does not change the truth status of Gödel's work within the context chosen by him. What we do with Gödel's theorems inside of normal mathematics is not altered since we do not there necessarily use the alternative logic of Colignatus, or anyone else for that matter.

I do believe that these researches are a beautiful antidote to the attempts which many people have made to draw grandiose conclusions from Gödel's work in completely different realms. For instance, assuming the consistency of the conventional formalism of mathematics, we know that the Gödel statement is true, and does not have a proof within that formalism. To me this merely shows that that formalism is rather restricted. We can do lots of number theory in that formalism, but not all of it. Whether or not there exists a finite set of axioms for number theory depends on how you define an axiom. As mathematicians we do mathematics by arguing about proofs, i.e., we prove theorems in mathematics by meta-mathematics, and it is one of the joys of mathematics that this is possible. Gödel showed in the thirties of the last century that what at that time appeared a hopeful candidate for capturing in formal language the whole essence of mathematics, did not do the job, and it suggests that the enterprise was in fact doomed to failure. I think this is a wonderful finding, a tribute to human creativity. I do not think however that Gödel's theorem tells us something about quantum physics or biology or theology.

However, the famous (and respected) Penrose uses Gödel's theorem to argue, in his book 'The emperor's new mind', that artificial intelligence can never be possible, because we can prove theorems which no computing device can prove. He goes on to suggest that this is connected to some so far unknown quantum mechanical phenomena in our neural system. Perhaps today he would argue that quantum computers will be able to 'think'.

This is not the only example of great minds of science and philosophy who have built incredible castles in the air on Gödel's theorem. Grand musings on philosophy and modern thought seem to have an obligatory discussion of the relevance of Gödel to this or that metaphysical position, but so far I have only seen these musings prove that the writer did not know much about what he or she was pontificating about. I think that Colignatus's book is a beautiful antidote to that kind of thinking.

There are some features of ALOE which I must describe to you before you rush to internet to download the pdf file. (I hope you will still do that, after I tell you about these features; but you will know in advance what to expect). The book has been conceived and developed as a Mathematica notebook. Colignatus's logic is implemented in an extension to the Mathematica language, which becomes accessible to you if you buy his 'Economics Pack', an extensive collection of Mathematica modules for teaching and research in economics and econometrics. If you already have access to Mathematica then there is no problem for you. Otherwise you will either have to get Mathematica, or to miss part of the fun of the book. Since ALOE has an original approach to logic you will not be surprised that the rest of the Economics Pack contains several original approaches to other parts of the author's field. His original innovations are in my opinion often thought provoking.

The book is presented as a course-book in logic for first year university or college students, apparently since the author feels that modern logic has to be built up again from bottom up. He also suggests that the use of a computer-algebra program helps us to become more familiar with three-valued logic. I cannot judge how succesful this would be (e.g. the book contains no explicit exercises). I do not work every day with Mathematica: every time I do have to use it I have to relearn the basic tricks and I get irritated by Wolfram's quirks. I am also irritated by the huge price and pretensions of this particular computer package. Hopefully there will eventually be an open source alternative (the SAGE project for instance...).

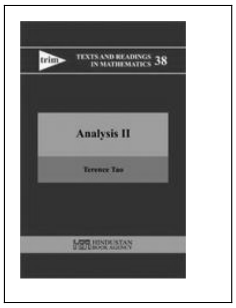
This means that the reader of ALOE who actually wants to play with the Mathematica examples has to somehow get hold of Mathematica, which for me always implies finding out again how to get the (my university's campus) licence key installed, a complicated process which is so unnatural that I cannot remember it from one year to the next. Secondly you have to install 'The Economics Pack' into Mathematica, and the instructions may be a bit unclear if like me you are not used to the process. After solving all these problems, the ALOE notebooks did work, but not all of them exactly as the book suggested they should. This concerned the format of the output, not the output itself, but it certainly adds to some irritation. Mathematica allows various ways of display: OutputForm, InputForm, StandardForm and TraditionalForm, and the user has to find out that the book uses TraditionalForm.

My other complaint about this book is a complaint which I have about much private modern desk-top-publishing. Firstly, an author of a mathematics textbook is not usually an expert in typography. Secondly, a Dutch author of an English language mathematics textbook is not usually a native speaker, and the book is full of little Hollandicisms and mistakes in grammar and spelling which a professional book editor would have removed before sending the manuscript on to the printers. The Hollandicisms are cute of course (though probably this was not the effect intended by the author), but the incidental spelling and grammar mistakes — don't get me wrong, the author writes lively and enjoyable and transparent English — are an irritation. I read a great deal and I enjoy reading professionally produced books. It irritates me to read the kind of mistakes in typography and language which I find myself daily pointing out to my students. It is not because these blemishes are mistakes, it is because bumping into them all the time distracts and slows down, the form gets in the way of the content. The point of good typography and good writing is that the form becomes almost invisible, except when of course it is part of the message. I understand that the author had the choice between publishing this book as it is or not publishing at all, and it is indeed better that the book is available, but a second edition would benefit from a polishing round.

As I said above, the author is a creative thinker. The present book started life out of interest during his econometrics education in Groningen around 1981, and has been lying on a shelf for more than 25 years, and was retyped and cast in Mathematica in 2007 after his return from a stay abroad when the manuscript turned up again. The author feels that his admittedly imperfect work, and at the start of his reading in logic also the work of an outsider and a novice, was not properly appreciated by his 1981 professors in mathematical logic. He in fact thanks them for their criticism that sharpened his thoughts and allowed ALOE to get its form but he also believes that they did not really listen. I contacted one of those professors and did observe that there was little communication indeed. This personal history does rear its head at various points in the book. Personally, I found the history adds to the drama and content of the book, and it is surely partly responsible for the energy and drive with which it is written. The author on his website emphasizes his analysis on unemployment, and one can imagine that had he received more attention in 1981 by logicians, then the directorate of CPB might have had a different reputation to deal with. I smile at some of the more far reaching claims of the author, I enjoy the positive and original content. The book contains much historical and anecdotal material on logic, selected from respectable sources on the history of mathematical logic, and evaluation from the author's personal perspective, which I enjoyed reading very much.

In conclusion I think this is a book which deserves to be read and to be discussed and to be improved; it needs to be read with an open mind and in a constructive mood. Then I think many readers will find it educational and thought-provoking.

Richard Gill



T. Tao

Analysis I, II

Texts and Readings in Mathematics, vol 37, 38
New Delhi, Hindustan Book Agency, 2006
Vol I: 420 p.; Vol II: 272 p. prijs $ 36, $ 29
ISBN 81-85931-62-3; 81-85931-63-1

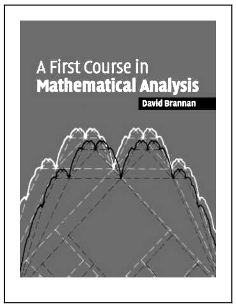
De analyseboeken van Fields-medaillewinnaar Terence Tao richten zich op formele bewijzen en rigoureuze afleidingen. Deze aanpak gaat ervan uit dat de analysecursus wordt gegeven naast (of na) een calculuscollege waarin gerekend kan worden aan reeksen, integralen, enzovoort. Het eerste deel van de twee boeken begint met een hoofdstuk waarin wordt uitgelegd wat analyse is en waarom we eigenlijk analyse willen doen. Dit hoofdstuk eindigt met de leerdoelen van de boeken met als afsluitende zin "You will develop a sense of *why* a rule in mathematics (...) works, how to adapt it to new situations, and what its limitations (if any) are; this will allow you to apply the mathematics you have already learnt more confidently and correctly". Het boek slaagt er inderdaad in om dit gevoel over te brengen.

Analysis I begint daarna met de natuurlijke getallen, verzamelingen, rationale getallen en de constructie van de reëele getallen in de eerste 140 pagina's. Afhankelijk van andere colleges in het curriculum kan een deel hiervan worden overgeslagen. Daarna wordt reëele analyse in één veranderlijke gedaan: rijen, reeksen, oneindige verzamelingen, functies, continuïteit, differentiatie en de Riemannintegraal. Dit beslaat ongeveer tweehonderd pagina's, de helft van het boek. De stijl van het boek is precies als het gaat om bewijzen, maar veel bewijzen van uitspraken worden als oefening aan de student gelaten. Het gros van de opgaven in het boek bestaat uit bewijsopgaven, waarbij vaak hints worden gegeven. De toon wordt informeler als er over de resultaten wordt gesproken. Dan wordt gepoogd de intuïtie voor een resultaat bij de lezer te vergroten. De lezer wordt direct aangesproken, en dit heeft geresulteerd in een zeer leerzaam boek voor een analysecursus. Het eerste deel heeft twee appendices. Er is een appendix over het decimale stelsel, en er is een appendix over redeneren in wiskundige bewijzen. Deze appendix geeft uitleg over bewijstechnieken: wat is een implicatie, wat is een bewijs uit het ongerijmde, structuren van bewijzen, kwantoren.

Om een college aan de hand van dit boek tot een succes te maken is het essentieel dat studenten opgaven maken. Daarom hebben we aan de Radboud Universiteit het inleveren van huiswerk en het geven van een groepspresentatie over een resultaat of opgave verplicht gesteld. Hierdoor worden de studenten gestimuleerd het boek goed te lezen, en dan blijkt het boek ook goed bij de belevingswereld van de studenten aan te sluiten. Zij hebben een positieve mening over het boek, en deze mening deel ik als docent.

In het deel Analysis II worden eerst metrische ruimtes, continue functies, uniforme convergentie en machtreeksen besproken. Daarna volgen Fourierreeksen, een onderwerp dat overgeslagen zou kunnen worden. De differentiaalrekening voor functies van meerdere veranderlijken met de impliciete en inverse functiestelling is het volgende standaardonderwerp. In plaats van de Riemannintegraal voor functies van meerdere veranderlijken te ontwikkelen, is gekozen voor een presentatie van de Lebesgue-integraal in $\mathbf{R}^n$. De integraalstellingen van Gauss, Stokes en Green, standaardonderdelen van een calculuscursus, worden niet besproken, en dat is een goede keuze.

Kortom, de boeken van Tao zijn zeer geschikt voor een analysecursus als deze naast (of na) een calculuscursus wordt gegeven. *Erik Koelink*



D.A. Brannan
A First Course in Mathematical Analysis
Cambridge University Press, 2006
459 p., prijs £25.99
ISBN 0-521-68424-2 (paperback)

In het hoger onderwijs wordt alles wat te maken heeft met functies, limieten, differentiëren en integreren opeens analyse of calculus genoemd. Een nieuw woord en een grote overgang, want opeens lijkt alles wat je als leerling eerder geleerd hebt vrij nutteloos. Je begint helemaal opnieuw en op een veel abstracter niveau. Bij veel universitaire opleidingen start men met 'dikke' studieboeken waarin differentiaal- en integraalrekening en toepassingen centraal staan. Mede door formeler taalgebruik wordt de overgang als lastig ervaren. Veel gebruikte boeken zijn de boeken van Adams en van Steward. In die boeken staan veel uitgewerkte voorbeelden die studenten houvast moeten bieden. Grondige bewijzen worden niet altijd gegeven of staan in appendices.

Dat geldt niet voor het boek *A First Course in Mathematical Analysis*. In dat boek wordt geen aandacht besteed aan toepassingen en het is dus vooral bedoeld voor studenten wiskunde. Het boek is gebaseerd op cursussen die ontwikkeld zijn aan de Open University (UK). Uitgangspunt is dat de leertekst geschikt is voor zelfstudie en dat uitleg daarom grondig moet zijn.

Omdat de auteur zich ervan bewust is dat analyse niet eenvoudig is, stelt hij de introductie van de gebruikelijk epsilon-delta 'horror' uit en kiest hij voor een opbouw met rijen: de sequential approach.

De opbouw van het boek is doordacht. In hoofdstuk 1 worden de reële getallen ingevoerd als decimale getallen: $\pm a_0, a_1 a_2 a_3 ...$ met $a_0 \in \mathbb{Z}$ en $a_1, a_2, a_3...$ cijfers uit $\{0, 1, ..., 9\}$. Een slimme zet want op die manier bewijst de auteur in hoofdstuk 1 eenvoudig dat elke niet-lege verzameling van $\mathbb{R}$ een kleinste bovengrens heeft. In dat eerste hoofdstuk worden manipulaties met ongelijkheden besproken die veelvuldig gebruikt zullen worden. Ook komen de ongelijkheden van Bernoulli en van Cauchy-Schwarz, de driehoeksongelijkheid, het begrip supremum en het bewijs uit het ongerijmde aan de orde. Over de rekenregels van $\mathbb{R}$ stapt de auteur lichtjes heen.

In hoofdstuk 2 worden rijen besproken en komt al snel 'de epsilon' aan de orde (zonder zijn maatje delta). Duidelijke voorbeelden laten zien hoe je bewijst dat een rij een nulrij is. De rekenregels van rijen worden bewezen en het hoofdstuk eindigt met de invoering van e als limiet van de rij $\left\{(1+\frac{1}{n})^n\right\}$, terwijl π wordt ingevoerd als de limiet van oppervlaktebenaderingen van de cirkel. Nauwkeurig wordt bewezen dat die rij convergeert. Na hoofdstuk 3 (reeksen) wordt in hoofdstuk 4 het begrip continuïteit ingevoerd met behulp van rijen. Verder vinden we in dat hoofdstuk bewijzen dat de sinus, de exponentiële functie en hun inversen continu zijn, en als afsluiting het bewijs van de tussenwaardestelling en de extreme-waarden-stelling.

In hoofdstuk 5 komt dan na de behandeling van continuïteit het limietbegrip en de lang uitgestelde ϵ, δ-definitie. Als motivatie voor dat uitstel wordt aangedragen dat het gedrag van een willekeurige convergente rij soms moeilijk te analyseren is. Veel voorbeelden laten zien hoe je bewijst dat een functie continu is. Als hoogtepunt volgt het bewijs dat de functie van Dirichlet discontinu is in elk punt. Ook in hoofdstuk 6, *Differentiation*, komt een 'vreemde functie' aan de orde: de functie van Blancmange (grafiek op de omslag van het boek), een functie die overal continu is maar nergens differentieerbaar. Een pittig bewijs volgt. Ook de stelling van Rolle, de regel van L'Hôpital en de middelwaardestelling worden in hoofdstuk 6 bewezen. In de laatste hoofdstukken komen integraalrekening en machtreeksen aan bod. Net als in de voorafgaande hoofdstukken wordt de theorie zeer zorgvuldig opgebouwd.

Het boek is voorzien van opgaven met uitwerkingen en ziet er verzorgd uit. Aanbevolen voor opleidingen die analyse meteen fundamenteel willen grondvesten. Volgens de auteur zijn er al meer dan tienduizend studenten (sinds 1971) die de cursus met succes bestudeerd hebben. Daarbij moet aangetekend worden dat Brannan eerlijk aangeeft dat het door allerlei curriculumveranderingen in Engeland niet eenvoudiger geworden is. Een experiment waard in Nederland? *Jan Essers*