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Freudenthal 100 symposium

Realistic Mathematics Education, past and present

Het wiskundeonderwijs richt zich al enige tijd veel te veel op toepassingen. Dit betoogt Erich Ch. Wittmann, (sinds 2004 emeritus) hoogleraar wiskundedidactiek aan de Universiteit Dortmund. In dit artikel schetst hij zijn eigen kijk op Freudenthals bijdrage aan de wiskundedidactiek en haar gevolgen. De ontwikkeling van 'Realistic Mathematics Education' ook in het na-Freudenthalse tijdperk heeft hij, sinds enkele jaren met groeiende scepsis, gevolgd. Erich Ch. Wittmann heeft Freudenthal goed gekend. Hij formuleert concrete voorstellen hoe de academische wereld er systematisch op in zou kunnen spelen.

Als onderzoeker in de didactiek van wiskunde geniet professor Wittmann bekendheid door zijn opvattingen over wiskundedidactiek als 'design science'. In het jaar 2000 heeft hij hierover op het '9th International Congress on Mathematical Education' (ICME 9) in Japan een plenaire voordracht gehouden. Reeds voor het wiskundeonderwijs op de basisschool legt hij zowel de nadruk op wiskundige diepgang als ook op bruikbaarheid in de praktijk. Samen met Gerhard N. Müller riep hij in 1987 het 'mathe 2000' project aan de Universiteit Dortmund in het leven. Via 'Das Zahlenbuch' en andere publicaties vanuit het project konden sindsdien generaties Duitse kinderen rechtstreeks van nieuwe wiskundig didactische inzichten profiteren. Ook is hij als wetenschappelijk adviseur betrokken bij het in heel Duitsland grootschalig opgezette SINUS-project. Voor zijn levenswerk onving hij in 1998 een eredoctoraat aan de Universiteit Kiel.

In 1967 the department of mathematics at the University of Erlangen organized an international colloquium in commemoration of the famous geometer K.G.Ch. von Staudt (1798–1867), who had worked as a professor at this university. Hans Freudenthal, an international expert also in the foundations of geometry, was one of the invited speakers, and I was eager to meet him for the following reason: I had just finished my studies as a prospective teacher of mathematics and physics for the Gymnasium level and was working on my doctoral dissertation in the theory of infinite groups as a research assistant at the Department of Mathematics in Erlangen. As I was seriously considering the option of moving into mathematics education at a later point of my career I had also started to read the literature in this rapidly growing field and while doing so I developed a strong aversion against the *New Math* movement, which at that time seemed to override the teaching of mathematics at both universities and schools. In this critical situation Freudenthal's paper 'What is axiomatics and what educational value can it have?', published in 1963 in *Der Mathematikunterricht*, was an enlightenment for me in several respects: The paper contained a convincing refutation of Bourbaki's architecture of mathematics as a basis for mathematics teaching. Moreover, the paper was written in a style I had never seen before: brilliant, witty and unconventional. For example, the hesitation of a mathematician to publish a paper according to its genesis was compared to the feelings of a man standing in the street in his underwear. Wow! The most important point, however, was the following: The paper described learning as a process that passes through different stages, each one a necessary step for the next one. The paper emphasized mathematical activity as the crucial element of learning and described 'local ordering' as a reasonable alternative to ready-made axiomatics.

At the colloquium I had a chance to talk to Hans Freudenthal for one hour, and here I learned of his fresh initiatives as the president of *International Commission on Mathematical Instruction* (ICMI): the foundation of an international journal in mathematics education (*Educational Studies in Mathematics Education* (ESM), first published in 1968), the organisation of the *First International Congress in Mathematics Education* (ICME 1) in Lyon in 1969, the establishment of a research group in mathematics education at the University of Utrecht (the *Instituut Ontwikke-*

ling Wiskundeonderwijs (IOWO), 1971). After this conversation I was fully convinced that mathematics education was 'my' field, and I was determined to follow Freudenthal's ideas.

In the following years Hans Freudenthal elaborated his vision in many articles and books, among them *Geometry Between the Devil and the Deep Sea* (presented to the Carbondale Conference on Teaching Geometry in 1969), *Mathematics as an Educational Task*, *Pupils' Achievements Internationally Compared — the IEA* (in ESM 1975), *Weeding and Sowing*, *Didactical Phenomenology of Mathematical Structures*, and finally *Revisiting Mathematics Education*, a kind of legacy.

What impressed and influenced me likewise was the work that Freudenthal initiated at the IOWO in Utrecht. Here he had gathered a team of highly creative mathematics educators, among them Aad Goddijn, Fred Goffree, Martin Kindt, Jan de Lange, Ed de Moor, Leen Streefland, George Schoemaker and Adri Treffers. The members of this team certainly stimulated him as much as he stimulated them. The special issue *Five Years of IOWO* (in ESM 1976) gives a lively account of what this group had achieved in the early seventies.

The developmental research conducted at the IOWO served as a model for our project *mathe 2000*, and so for good reasons Hans Freudenthal is one of the four arch fathers of *mathe 2000*. I wish I had been a member of the IOWO team in the early seventies. This lack was in part compensated for in the eighties when I was lucky to join Hans Freudenthal as a member of the editorial board of the journal *mathematik lehren*. For me the frequent weekend meetings of this board were like in-service courses across the unbelievably wide variety of topics that this great scholar mastered. As a rule the discussions were continued in the publisher's bar and went far into the night.

What is special about Freudenthal's approach? How do present developments in mathematics and mathematics education appear in the light of his principles?

photograph: Konrad Jacobs

Freudenthal at the von Staudt colloquium (spring 1967)

Mathematics as an educational task

Hans Freudenthal has taught us to look at mathematics as a field of knowledge that is firmly integrated into our culture and determined by both external ('applied') and internal ('pure') factors. His masterpiece *Mathematics as an Educational Task* bears witness to this conviction. Richness of relationships (Beziehungshaltigkeit) was a postulate to which he continuously referred, and it included both structural relationships and relationships with the real world. Today the cultural diversity of mathematics is endangered by both a growing specialisation within mathematics and a too strong swing of the pendulum towards applications. Departments of mathematics are less and less perceived as intellectual centres from which an orientation for mathematical education beyond narrow disciplinary boundaries is expected. Universally educated scholars like Hans Freudenthal who are able to communicate with the public are badly missing. It is quite typical that the German mathematicians needed an essayist, Hans Magnus Enzensberger, to speak for them in a famous article *Zugbrücke außer Betrieb* (drawbridge closed) published in a big newspaper. The problem could be mitigated to some extent by collective efforts of all members of a department or of a mathematical association. But as experience shows systematic and long-term concerted actions are hard to establish because of the growing specialization and of the 'ideology of self-restriction' (Roland Fischer). Many specialists dont even notice that something fundamental has gone wrong. The long-term consequences of this cultural vacuum are hard to predict. Possibly we have here one of the major reasons why less and less young people are attracted to the study of mathematics.

A genetic view of teaching and learning

In Freudenthal's view the learner has no choice but to 're-invent' mathematics under appropriate guidance by starting as a child from most elementary experiences and managing more and more complex structures with growing expertise. Mathematical knowledge can never be transmitted top-down in a ready-made form. Even the most perfect lecture can become vital for a student only if he or she makes sense of it by actively re-constructing in personal terms what has been proposed. Hans Freudenthal radically objected to the idea of a didactical transposition from the level of specialists to lower levels. In his talk at the *Carbondale Conference on Geometry* he put this view in his typical language:

"Geometry is endangered by dogmatic ideas on mathematical rigor. They express themselves in two different ways: absorbing geometry in a system of mathematics like linear algebra, or strangulating it by rigid axiomatics. So it is not one devil menacing geometry as suggested in the title of my paper. There are two. The escape that is left is the deep sea. It is a safe escape if you have learned swimming. In fact, that is the way geometry should be taught, just like swimming."

Again, the present situation at universities is not favourable. Prospective teachers are rarely given a chance to look at mathematics from a genetic perspective in a systematic way as most teacher education programs are organized like courses for specialists or even composed of courses for specialists (Analysis I and II, Linear Algebra I and II, ...). This is a serious problem.

Mathematics education as a research domain

In the preface of *Five Years IOWO* Hans Freudenthal stated: "IOWO is not a research institute; its members do not regard themselves as researchers but as producers of instruction, as engineers in the educational field, as curriculum developers. Engineering needs background research and can produce research as fall-out. Though both of them will be visible in the present account, its nucleus is our productive work, represented by a few specimens, and embodies our views on mathematics as a human activity and on curriculum development as a classroom activity, guided by curriculum developers, in close contact with all those interested in mathematics education."

A few years later he still looked sceptically at mathematics education as a research field, but nevertheless wrote his *Weeding and Sowing* as a kind of prologue for an emerging research field. In this book he clearly separated the research he had in mind from the research on teaching and learning that is conducted by psychologists, pedagogues, sociologists and other generalists who do not and cannot take the content properly into account. A decade later in *Revisiting Mathematics Education* he described more precisely the developmental research which he thought appropriate for a science of mathematics education. This view is closely related to the conception of mathematics education as a 'design science' which was developed by *mathe 2000* and which the former members of the IOWO explicitly shared. In the meantime, however, international mathematics education as a whole has certainly not moved into the direction that Freudenthal had proposed in his writings. As the two volumes of the ICMI-study *Mathematics Education as a Research Domain. A Search for Identity* show, the bulk of research in mathematics is too much dominated by general educational theories, does not pay enough attention to mathematical content and is far remote from the teaching practice. It is ironic that some part of recent research conducted at the institute carrying Freudenthal's name also falls into this category.

The loss of mathematical substance in mathematics teaching

The curricular conception of mathematics teaching developed at the IOWO has been called *Realistic Mathematics Education* (RME). At that time it was quite appropriate to emphasize the new curricular elements that the IOWO introduced in contrast with *New Math*, namely the relationships of mathematics with the real world. Structural aspects of mathematics nevertheless remained a firm part of RME in the early seventies. In Adri Treffers' doctoral dissertation *Three Dimensions* written under Freudenthal's supervision, 'horizontal' mathematisation, related to the applied aspect of mathematics, and 'vertical' mathematisation, related to the pure aspect, were clearly delineated. Later this balance became more and more distorted. The influence of RME at the international level has been enormous. However, for this seeming success a very high price had to be paid. While spreading almost over the whole world RME was exposed to developments which had their own momentum: to research in mathematical education which had lost the connection with mathematics, to a one-sided orientation towards superficial 'applications', and to the testing movement which replaced 'contents' by lists of 'competencies'. Step by step mathematical substance was pushed into the background and got lost. Particularly as far as the secondary level is concerned, the educational system has given its own meaning to the original conception of RME. The result is a kind of 'RME light' which can less and less guarantee a sufficient preparation for academic studies. This might be another major reason for the declining numbers of students who are interested in mathematics or science.

What about the future?

Hans Freudenthal's conceptions of mathematics and mathematics education are still valid. However, over the past twenty years there have been developments at the universities and in society that have not been favourable for bringing these conceptions to bear. The present situation is neither good for mathematics nor for mathematics education. A return to sound conditions requires first and foremost a reconstruction of the disciplinary organization of universities. My proposal is to found mathematical departments *within faculties of education* as some Asian countries have done, though somewhat half-heartedly. These departments could concentrate on teacher education (and possibly on the education of engineers, computer scientists, et cetera) and design novel programs which are less oriented towards presenting polished theories but more towards developing rich elementary mathematical theories in a research-like atmosphere where mathematicians do not hesitate to step before students in their mathematical underwear! This construction would result in more influence of mathematicians on mathematics education and at the same time in more influence of educators and teachers on mathematicians — to the benefit of both sides. Specialisation is a common feature in mathematics. So it is only natural to have mathematicians who are specializing in education. After all it is education that is of utmost importance for the welfare of societies.

This is not to say that approaches to mathematics education anchored in other disciplines and in the teaching practice are less important. Mathematics education is an interdisciplinary field which can develop properly only with contributions from many sources and with a clear orientation towards the teaching practice. In this polyphonic concert the voice of mathematics must nevertheless be leading. However, as Hans Freudenthal's work shows, mathematics can play this leading role only if it is conceived of as an integral part of culture, if the importance of its elementary parts is clearly recognized and if a genetic view of teaching and learning is adopted.