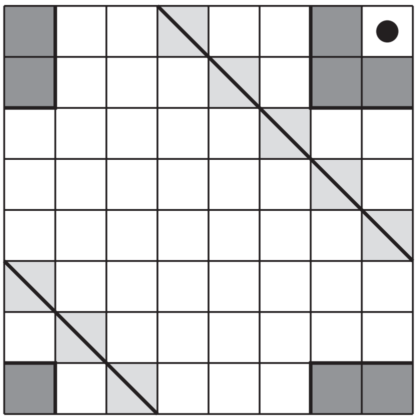
Problem

| Problem Section

This Problem Section is open to everyone; everybody is encouraged to send in solutions and propose problems. Group contributions are welcome. We will select the most elegant solutions for publication. For this, solutions should be received before **15 October 2021**. The solutions of the problems in this issue will appear in one of the subsequent issues.

Problem A (proposed by Daan van Gent)

Write $T=(\mathbb{Z}/8\mathbb{Z})^2$ for the *torus chessboard*. For every square $t\in T$ its *neighbours* are the squares in the set $\{t+d \mid d\in D\}$ for $D=\{-1,0,1\}^2\setminus\{(0,0)\}$. A *line* is a set of squares of the form $\{t+nd \mid n\in\mathbb{Z}\}$ for $t\in T$ and $d\in D$. The following figure gives an example of the neighbours (coloured dark grey) of the dotted square, as well as an example of a line (coloured light grey). Disprove or give an example: There exists a pairing of neighbouring squares, i.e. a partitioning of T into sets $\{s,t\}$ of size 2 such that s and t are neighbours, such that every line contains a pair.



Problem B (proposed by Hendrik Lenstra)

Write φ for the Euler totient function. Determine all infinite sequences $(a_n)_{n\in\mathbb{Z}_{\geq 0}}$ of positive integers satisfying $\varphi(a_{n+1})=a_n$ for all $n\geq 0$.

Problem C (proposed by Hendrik Lenstra)

Let R be a ring. We say $x\in R$ is a *unit* if there exists some $y\in R$ such that $xy=yx=1$, and $x\in R$ is *idempotent* if $x^2=x$. Show that if every unit of R is central, then every idempotent of R is central.

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