Problemen

| Problem Section

This Problem Section is open to everyone; everybody is encouraged to send in solutions and propose problems. Group contributions are welcome. We will select the most elegant solutions for publication. For this, solutions should be received before **15 April 2020**. The solutions of the problems in this issue will appear in the next issue.

Problem A (proposed by Hendrik Lenstra)

For every positive integer n, we write $[n] := \{0, 1, \dots, n-1\}$. For every integer m, we let $T(m) := m(m+1)/2$ be the m-th triangular number. Let $\tau : [n] \to [n]$ be the map given by $m \mapsto T(m)$ mod n.

a. For which n is τ a permutation?
b. For these n, determine the sign of T as a function of n.

Problem B (proposed by Onno Berrevoets)

Let G be a finite group of order n. A map $f : G \to \mathbb{R}$ is called a near-homomorphism if for all $x, y \in G$, we have $|f(xy) - f(x) - f(y)| \leq 1$.

a. Show that for every near-homomorphism f from $G \to \mathbb{R}$, we have $\mathrm{diam}(f[G]) := \sup_{x,y \in G} |f(x) - f(y)| \leq 2 - 2/n$.
b. Show that if G is cyclic, then there exists a near-homomorphism $f : G \to \mathbb{R}$ with $\mathrm{diam}(f[G]) = 2 - 2/n$.

Problem C (proposed by Hendrik Lenstra)

Let $n \geq 4$ be an integer and let A be an abelian group of order 2^n. Let σ be an automorphism of A such that the order of σ is a power of 2. Then the order of σ is at most 2^{n-2}.

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