# Problemen

**| Problem Section**

This Problem Section is open to everyone; everybody is encouraged to send in solutions and propose problems. Group contributions are welcome. We will select the most elegant solutions for publication. For this, solutions should be received before **15 October 2019**. The solutions of the problems in this issue will appear in the next issue. Problem C, part b is a star exercise; we do not know a solution to this problem.

**Problem A** (proposed by Hendrik Lenstra)

Let $\tau : \mathbb{Z}_{>0} \to \mathbb{Z}_{>0}$ be the map such that $\tau(n)$ is the number of positive divisors of $n$ for any $n \in \mathbb{Z}_{>0}$. Show that there are uncountably many maps $f : \mathbb{Z}_{>0} \to \mathbb{Z}_{>0}$ such that $f \circ f = \tau$.
Note: This is a follow-up to Problem A of the March 2018 edition.

**Problem B** (proposed by Daan van Gent)

Let $X$ be a set and $* : X^2 \to X$ a binary operator satisfying the following properties:
1. $(\forall x \in X)\ x * x = x$;
2. $(\forall x,y,z \in X)\ (x * y) * z = (y * z) * x$.

Show that there exits an injective map $f : X \to 2^X$ that for all $x,y \in X$ satisfies $f(x * y) = f(x) \cap f(y)$.

**Problem C** (proposed by Onno Berrevoets)

a. Does there exist an infinite set $X \subset \mathbb{Z}_{>0}$ such that for all pairwise distinct $a,b,c \in X$ and all $n \in \mathbb{Z}_{>0}$ we have $\gcd(a^n + b^n, c) = 1$?

b.* Does there exist an infinite set $X \subset \mathbb{Z}_{>0}$ such that for all pairwise distinct $a,b,c,d \in X$ and all $n \in \mathbb{Z}_{>0}$ we have $\gcd(a^n + b^n + c^n, d) = 1$?

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