Problemen

| Problem Section

Problem A

Let $n > 2$ be an odd integer and let C be an embedding of the circle in $\mathbb{R}^n$. That is, $C = f([0,1])$, where $f : [0,1] \to \mathbb{R}^n$ is continuous, $f(0) = f(1)$, and f is injective on $[0,1)$. Show that there is an affine hyperplane in $\mathbb{R}^n$ that contains at least $n+1$ points from C.

Problem B

Place coins on the vertices of the lattice $\mathbb{Z}^2$, all showing heads. You are allowed to flip coins in sets of three at positions $(m,n),(m,n+1)$ and $(m+1,n)$ where m and n can be chosen arbitrarily. Is it possible to achieve a position where two coins are showing tails and all others show heads using finitely many moves?

Problem C

Say that a natural number is k-repetitive if its decimal expansion is a concatenation of k equal blocks. For instance, 1010 is 2-repetitive and 666 is 3-repetitive. Let R_k be the set of all k-repetitive numbers. Determine its greatest common divisor.

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