Problemen

Problem Section

Problem A (proposed by Gabriele dalla Torre)

Show that there are infinitely many primes that divide at least one integer of the form

$$2^{n^3+1} - 3^{n^2+1} + 5^{n+1}.$$

Problem B (proposed by Jinbi Jin)

Let n be a positive integer. Two players, Ann and Bill, play the following game. First, Ann distributes a number of balls over boxes numbered from 1 up to n. Then Bill chooses one of the boxes, and adds a ball to it. Finally, Ann attempts to empty all boxes, using only the following moves.

- Taking one ball from three consecutive boxes.
- Taking three balls from one box.

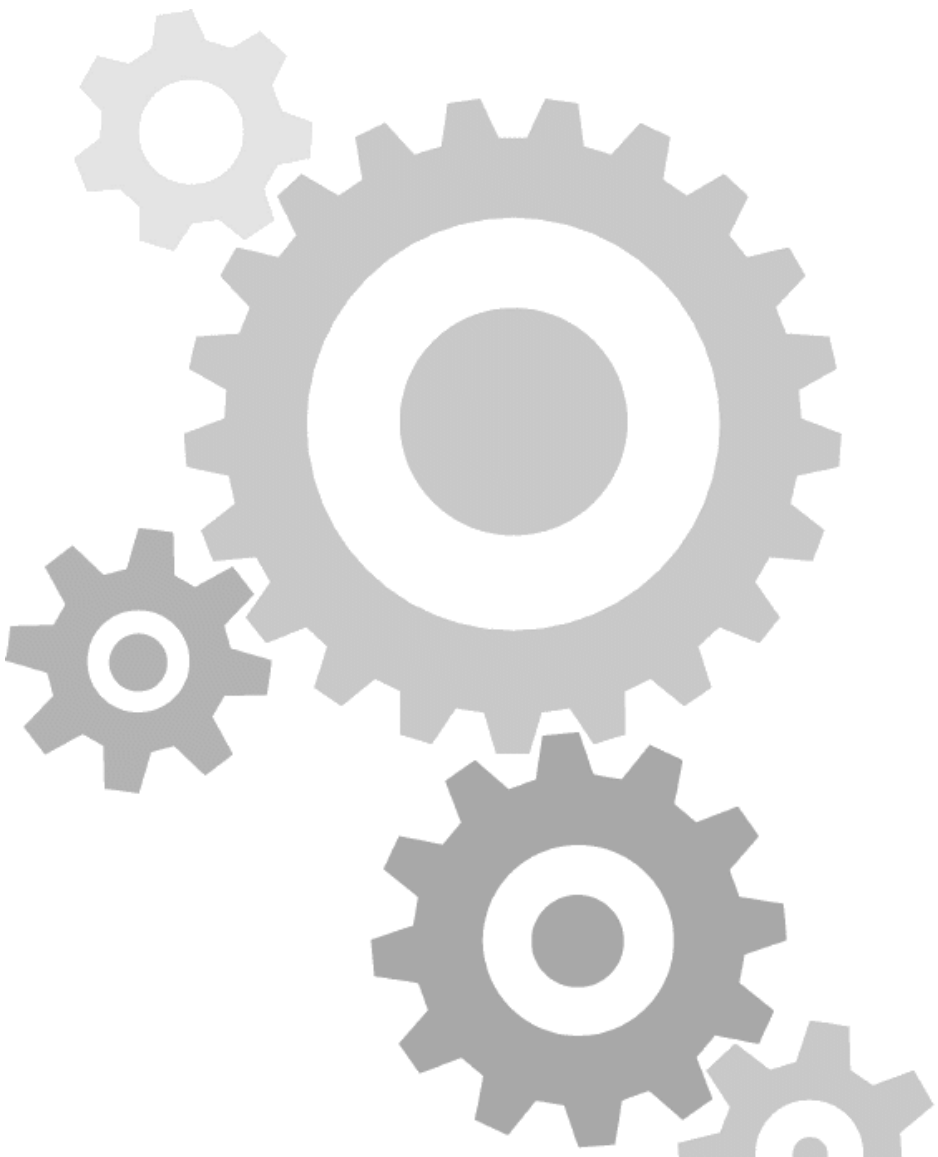
Ann wins if she succeeds in doing so, otherwise Bill wins.

1. Determine (as a function in n) the maximum number of losing moves Bill can have. What is the minimum number of balls Ann needs to attain this number?
2. Do the same as in point 1, if Ann in addition is allowed *only once* to remove two balls from one box.

Problem C (proposed by Hendrik Lenstra)

Let p be a prime number and let k be a positive integer. Prove that for every integer n there exist integers w, x, y, z such that

$$n \equiv w^p + x^p + y^p + z^p \mod p^k.$$



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