Problemen

Problem Section

Problem A (proposed by Jan Turk)

Let $k > 3$ be an integer. Determine the variance of the greatest common divisor of k positive integers. Here we mean the limit, as $n \to \infty$, of the variance of the greatest common divisor of k integers in $\{1, 2, \ldots, n\}$ with respect to the uniform distribution on $\{1, \ldots, n\}^k$.

Problem B (folklore)

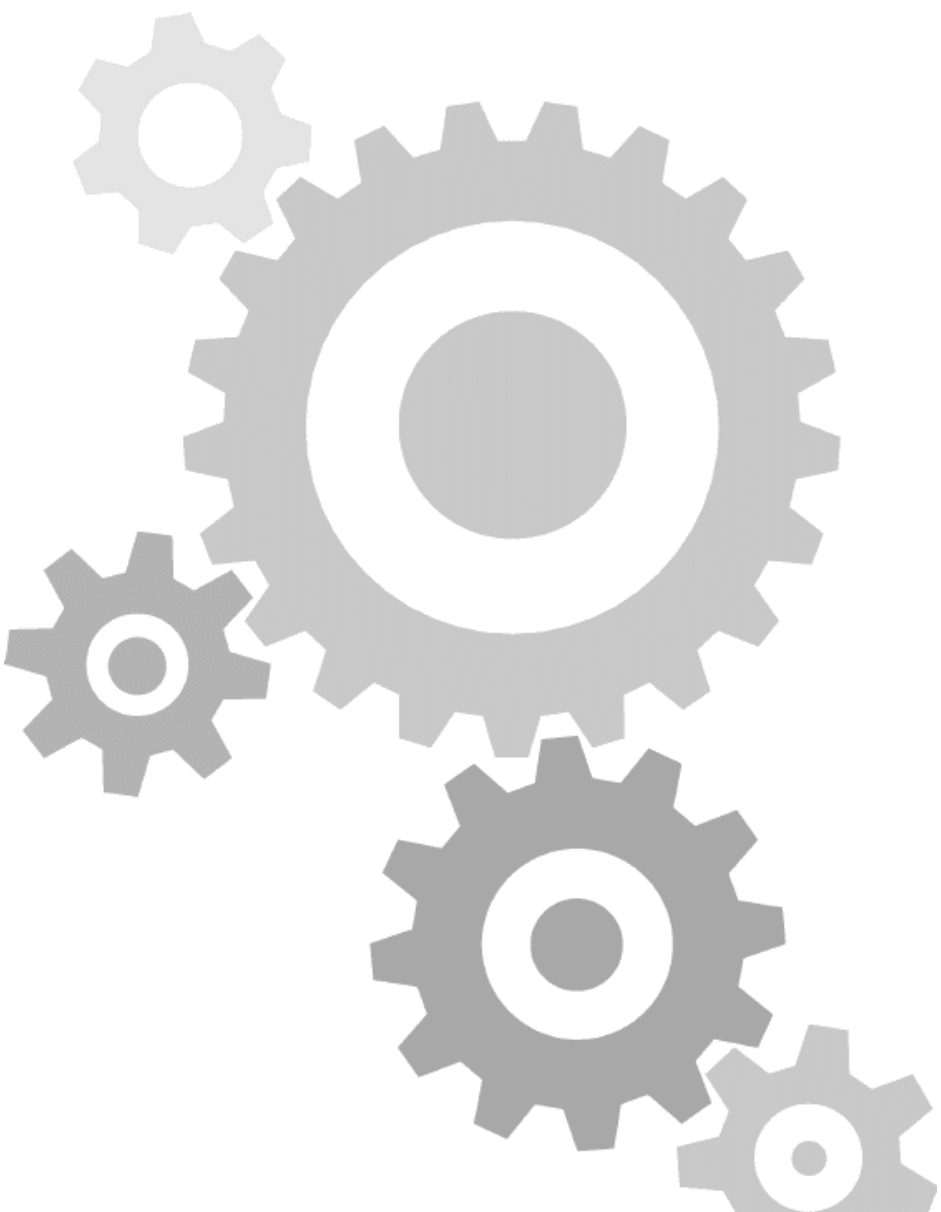
The evil Eve has locked Alice and Bob in a room without windows. Outside the room, there is a corridor with 64 doors. Eve puts a key behind one of the doors and a crocodile behind each of the others. Then she hangs up a light bulb above each of the doors, and for each light bulb, switches it on or off. Then Eve brings Alice into the corridor, tells her which door hides the key and tells her to choose one of the light bulbs and change the state of that chosen light bulb. After Alice leaves, Eve brings Bob to the corridor, and tells him to open a door of his own choice. Alice and Bob are allowed to discuss a strategy before Alice is shown where the key is, but not after.

a. Give a strategy that guarantees Bob to find the key.
b. For which positive integers n does such a strategy exist if there are n doors?

Problem C (folklore)

Let X_3 be the collection of three-element subsets of $\{1, 2, \ldots, 8\}$, and let X_4 be the collection of four-element subsets of $\{1, 2, \ldots, 11\}$. Does there exist an injective map $\phi\colon X_3 \to X_4$ with the following properties?

1. For all subsets $V \subseteq X_3$, we have $\#(\bigcap_{v \in V} \phi(v)) \geq \#(\bigcap_{v \in V} v)$.
2. For all $v, v' \in X_3$, if $v \cap v' = \emptyset$, then $\phi(v) \cap \phi(v') = \emptyset$.



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