Problemen

Problem Section

Problem A (proposed by Hendrik Lenstra)

Let G be a group, and let $a, b \in G$ be two elements satisfying $\{gag^{-1} : g \in G\} = \{a, b\}$. Prove that for all $c \in G$ one has $abc = cba$.

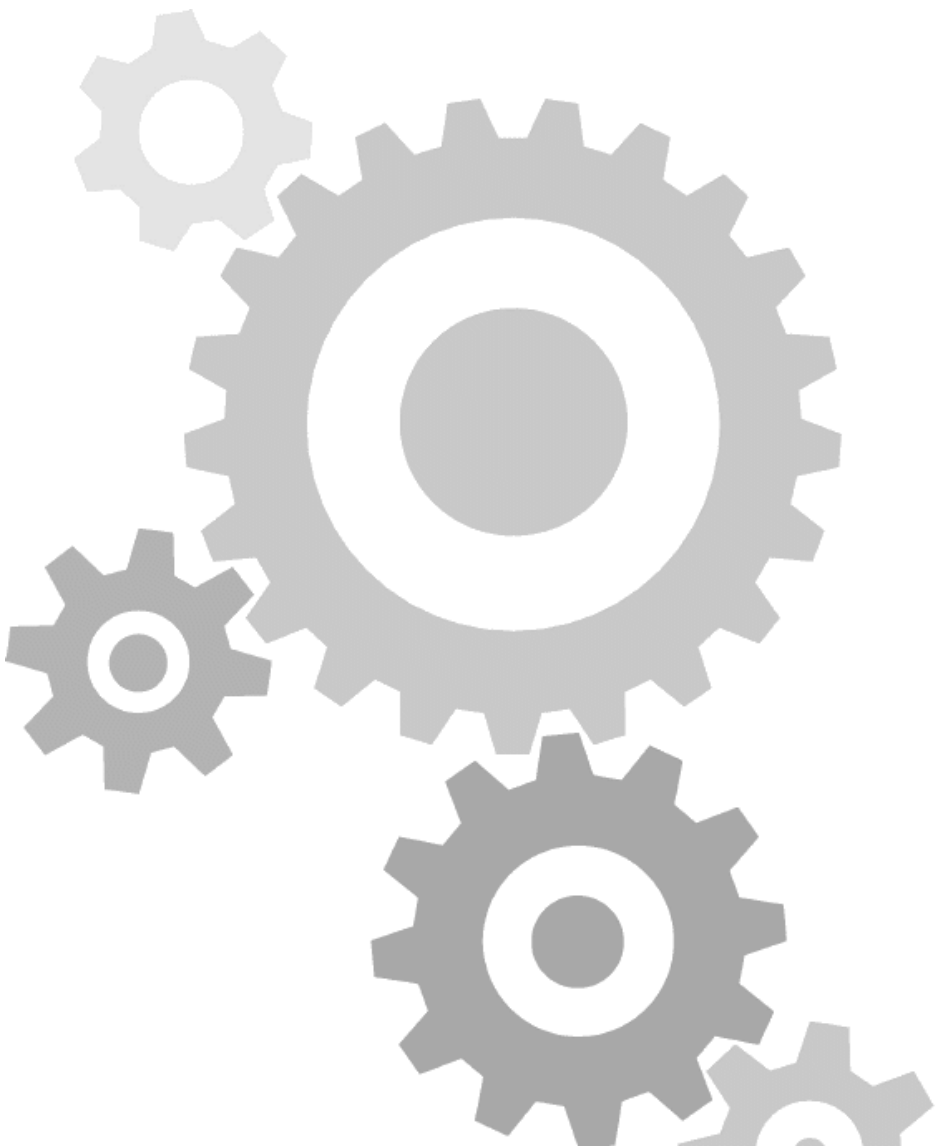
Problem B (the attribution will appear in the September issue of 2014)

Let K be a field, and consider for all positive integers n the subset S_n of $x \in K^*$ that can be written as the sum of n squares in K. Show that the subgroup of K^* generated by S_n is equal to $S_{t(n)}$. Here, for a positive integer n, we denote by $t(n)$ the smallest power of two that is greater than or equal to n.

Problem C (folklore)

Given five pairwise distinct points A, B, C, D, E in the plane, no three of which are collinear, and given a line l in the plane not passing through any of the five points. Assume that l intersects the conic section c passing through A, B, C, D, E. Construct the intersection points of l and c. One of the solutions which uses the smallest number of moves will be awarded the book token. Here, given a collection of points, lines, and circles in the plane, a *move* consists of adding to the collection either a line through two of the points, or a circle centered at one of them and passing through another. At any time one is allowed to freely add any intersection point among the lines and circles, as well as any sufficiently general point, either in the plane, or on any of the lines or circles.

For example, given a line ℓ and a point P on ℓ one can construct a line through P and perpendicular to ℓ in three moves as follows. Choose a point M not on ℓ. For the first move, take the circle C centered at M and going through P. Let Q be the second point of intersection between C and ℓ. For the second move, add the line through Q and M and let R be the second point of intersection between this line and C. Finally, add the line PR, which is perpendicular to ℓ.



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