Problemen

| Problem Section

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Problem A (proposed by Gerard Renardel de Lavalette)

We have two hourglasses, A for a seconds and B for b seconds, where a and b are relatively prime integers and $0 < a < b$. Let t_0 be an integer with $t_0 \geq b + (\frac{1}{2}a - 1)^2$. Show that A and B can be used to identify the time $t = t_0$ if the upper bulbs are empty at $t = 0$.
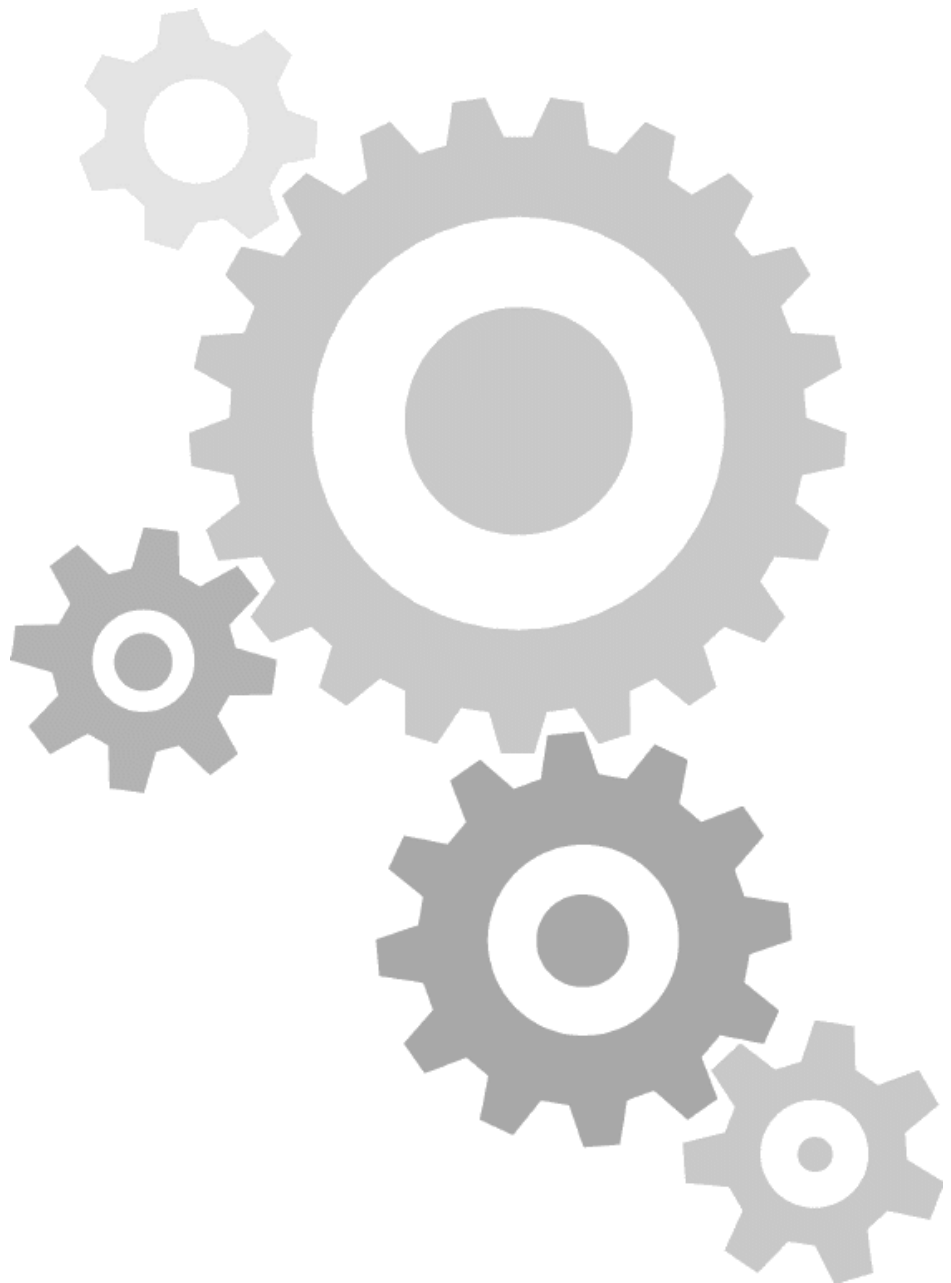
Problem B (folklore)

In a two-player game, players take turns drawing a number of coins from a pile that starts with n coins. The first player takes at least one coin from the pile, but not all. In the subsequent turns, each player takes at least one coin, and at most twice the number of coins taken in the previous turn. The player who takes the last coin wins.

For which numbers n can the first player win?

Problem C (proposed by Bas Edixhoven and Maarten Derickx)

Let $ABCD$ be a convex quadrilateral inside a plane U in $\mathbb{R}^3$. Suppose that $ABCD$ is not a parallelogram. Show that there exist a plane V in $\mathbb{R}^3$ and a point $P \in \mathbb{R}^3 - (U \cup V)$ such that if a light source is placed in P, then the shadow of $ABCD$ on V is a square.