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Problemen

Problem Section

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Problem A (folklore)

Let S be an open subset of $\mathbb{R}_{>0}$ that contains arbitrarily small elements. Prove that every positive real number can be written as a sum of finitely many elements of S.

Problem B (proposed by Gabriele Dalla Torre)

Let n be a positive integer. Show that every sequence of n elements of $\{0, \ldots, 9\}$ occurs as a sequence of consecutive digits in the last 2n digits of the decimal representation of some power of 2. Also, determine all $\alpha \in \mathbb{R}_{>0}$ for which the statement still holds if we replace 2n by $\lceil \alpha n \rceil$.

Problem C (proposed by Pietro Vertechi)

Let p be a prime number. Determine the smallest integer d for which there is a monic polynomial f of degree d with integer coefficients such that p^{p+1} divides f(n) for all integers n.

