Problemen

Problem Section

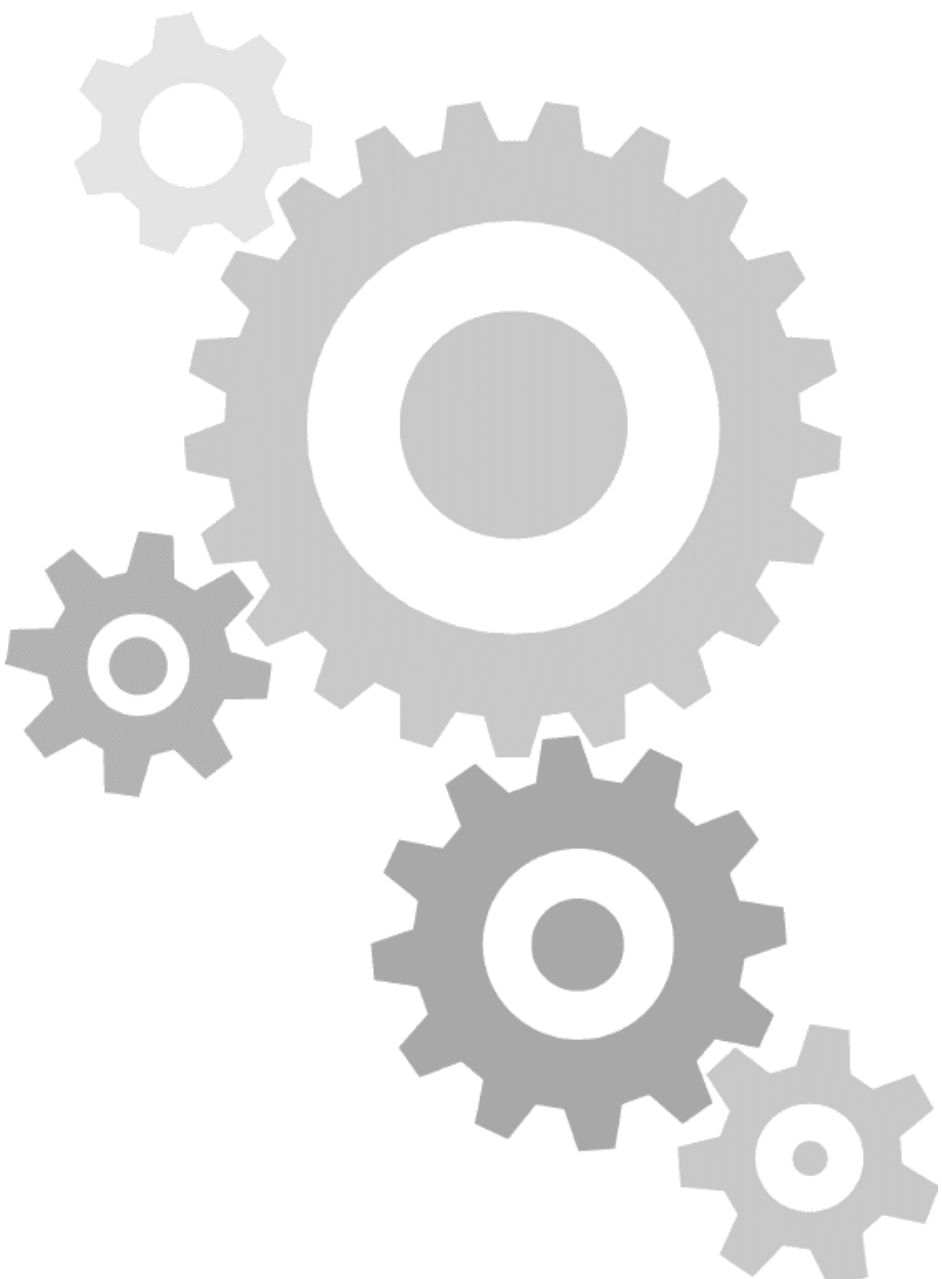
Problem A (proposed by Gabriele Dalla Torre)
Show that there are infinitely many prime numbers p for which there is a positive integer n with

$$2^{n^2+1} \equiv 3^n \pmod{p}.$$

Also, show that there are infinitely many prime numbers p for which there is no such n.

Problem B (communicated by Marten Wortel)
Let $f\colon \mathbf{R} \to \mathbf{R}$ be a continuous function that has a local minimum or maximum at every point of $\mathbf{R}$. Show that f is constant.

Problem C (communicated by Arne Smeets)
Let $f\colon \mathbf{Q} \times \mathbf{Q} \to \mathbf{Q}$ be a function such that for all $a \in \mathbf{Q}$ the functions $x \mapsto f(a,x)$ and $x \mapsto f(x,a)$ are polynomial functions from $\mathbf{Q}$ to $\mathbf{Q}$. Is it true that f is given by a polynomial in two variables? What if we replace $\mathbf{Q}$ by $\mathbf{R}$?



Redactie:
Johan Bosman
Gabriele Dalla Torre
Ronald van Luijk
Lenny Taelman

Problemenrubriek NAW
Mathematisch Instituut
Universiteit Leiden
Postbus 9512
2300 RA Leiden
problems@nieuwarchief.nl
www.nieuwarchief.nl/problems