Problem A (folklore)

Let k be a non-negative integer. Let $S \subset \mathbf{Z}$ be a set consisting of $2^{k+1} - 1$ integers. Show there exists a subset $T \subset S$ of cardinality 2^k such that the sum of the elements of T is divisible by 2^k.

Problem B (folklore)

Find all functions $f : \mathbf{R}_{>0} \to \mathbf{R}_{>0}$ such that

$$f(x + y) \geq f(x) + y f(f(x))$$

for all x and y in $\mathbf{R}_{>0}$.

Problem C (folklore)

Let V be an infinite-dimensional vector space. Show that the dimension of the dual space V^* equals the cardinality of V^*.

Submission deadline: **December 1, 2009**