Problem A (proposed by Hendrik Lenstra)

Let a and b be integers. Show that the following are equivalent:

- n divides $a^n - b^n$ for infinitely many positive integers n,
- $|a - b| \neq 1$.

Problem B (based on a problem by Hendrik Lenstra)

Let G be a finite group and a be an element of G. Show that the number of elements $g \in G$ that satisfy both $ga \neq ag$ and $ga^2 = a^2g$ is divisible by 4.

Problem C (folklore)

Let p be an odd prime number and A and B two $n \times n$ matrices with entries in $\mathbf{Z}$ such that $A^p = B^p = 1$, and such that the rank of $A - B$ is 1. Show that $n \geq p$.