Problemen

Problem Section

Problem A (proposed by Ovidiu Furdui)

Denote the fractional part of a positive real number x by $\{x\}$, for example $\{\pi\} = \pi - 3$. Evaluate the following double integral:

$$\int_0^1 \int_0^1 \left\{\frac{x}{y}\right\}\left\{\frac{y}{x}\right\} dxdy.$$

Problem B (Folklore)

Let S be a set consisting of 15 integers, and such that for all $s \in S$ there exist $a, b \in S$ with $s = a + b$.

1. Show that there exists a non-empty subset $T \subset S$ of at most seven elements that add up to 0.
2. Show that this does not need to be true for S with 16 elements.

Problem C (Proposed by the Arithmetic Geometry group of Leiden University)

Let $f : \mathbf{R} \to \mathbf{R}$ be a C^∞ function (that is, all higher derivatives of f exist and are continuous) such that

1. $f(x) = 0$ if $x \leq 0$,
2. $f(x) > 0$ if $x > 0$.

Is it true that $\sqrt{f} : \mathbf{R} \to \mathbf{R}_{\geq 0}$ is a C^1 function (that is, that its derivative exists and is continuous)?

Eindredactie:
Lenny Taelman, Johan Bosman
Redactieadres:
Problemenrubriek NAW
Mathematisch Instituut
Postbus 9512
2300 RA Leiden
uwc@nieuwarchief.nl